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PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

Date : 26 February, 2021 (SHIFT-1) Time ; (9.00 am to 12.00 pm)

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SUBJECT : MATHEMATICS

A-10 Road No. 1, IPIA, Kota-324005 (Rajasthan), India
Tel. : + 91-744-2665544 | Website : www.reliablekota.com | E-mail: info@reliablekota.com

MATHEMATICS

SECTION-A

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4\,\vec{b}$ (3) $\vec{a}\times\vec{b}$ (4) $|\vec{a}|^4\,\vec{b}$

Ans. (4)

Sol. $\vec{a}\times(\vec{a}\times((\vec{a}\,.\,\vec{b})\,\vec{a}-|\vec{a}|^2\vec{b}))$

$= \vec{a}\times(-|\vec{a}|^2(\vec{a}\times\vec{b})) = --|\vec{a}|^2\,((\vec{a}\,.\,\vec{b})\,\vec{a}-|\vec{a}|^2\vec{b}) = -|\vec{a}|^4\,\vec{b}-|\vec{a}|^2(\vec{a}\,.\,\vec{b})\,\vec{a}$

$= |\vec{a}|^4\,\vec{b}$ $(\because\ \vec{a}\,.\,\vec{b}=0)$

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{14}}$

Ans. (1)

Sol. $^nC_9\times\left(\frac{1}{2}\right)^9\times\left(\frac{1}{2}\right)^{n-9} = {}^nC_7\times\left(\frac{1}{2}\right)^7\times\left(\frac{1}{2}\right)^{n-7}$

$^nC_9 = {}^nC_7 \Rightarrow n = 16$

$P(2\text{Heads}) = {}^{16}C_2\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^{14}$

$= {}^{16}C_2\times\left(\frac{1}{2}\right)^{16}$

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

(1) 4 (2) 1 (3) 6 (4) 12

Ans. (1)

Sol. $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$A^2 = \begin{bmatrix} a^2+b^2 & b(a+c) \\ b(a+c) & b^2+c^2 \end{bmatrix}$

$tr(A^2) = a^2 + 2b^2 + c^2 = 1$

$\Rightarrow$ b = 0 and $a^2 + c^2 = 1$

$\Rightarrow$ (a, c) ≡ (1, 0), (–1, 0), (0, 1), (0, –1)

4. In a increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to

(1) 30 (2) 26 (3) 35 (4) 32

Ans. (3)

Sol. $ar + ar^5 = \frac{25}{2}$ and $ar^2 \cdot ar^4 = 25$ $\Rightarrow ar^3 = 5$

$\therefore \frac{r + r^5}{r^3} = \frac{5}{2}$

$\Rightarrow 2 + 2r^4 = 5r^2$

$\Rightarrow 2r^4 - 5r^2 + 2 = 0$

$\Rightarrow r^2 = 2$ or

$r^2 = \frac{1}{2}$ Reject

Now, $ar^3 + ar^5 + ar^7 = 5 + ar^5(1 + r^2) = 5 + 5.2(1 + 2) = 35$

5. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]} dx$ where [x] is the greatest integer $\le x$, is

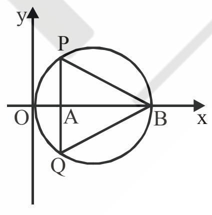
(1) 100(e – 1) (2) 100(1 – e) (3) 100e (4) 100 (1 + e)

Ans. (1)

Sol. $\sum_{n=1}^{n=100} \int_{n-1}^{n} e^{\{x\}} dx$

$= 100 \int_0^1 e^x = 100(e - 1)$

6. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$ (3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Ans. (2)

Sol. OB = 13

radius of circle = $\frac{13}{2}$

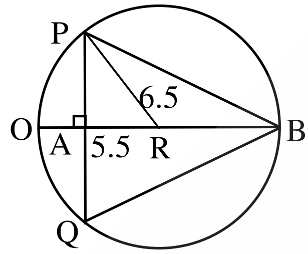
AB = OB – OA = 12

Let centre of circle is R.

AR = OR – OA = $\frac{13}{2}-1=\frac{11}{2}$

$(AP)^2 + (AR)^2 = (PR)^2 \Rightarrow (AP)^2 + \frac{121}{4}=\frac{169}{4} \Rightarrow (AP)^2 = 12$

$(AP) = 2\sqrt{3} \Rightarrow PQ = 2\left(2\sqrt{3}\right)=4\sqrt{3}$



Area of $\Delta PQB = \frac{1}{2}\left(4\sqrt{3}\right)(12) = 24\sqrt{3}$

7. The sum of the infinite series $1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+...$ is equal to

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

Ans. (1)

Sol. $S = 1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+......\infty$ (i)

$\frac{1}{3}S = \frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+..........$ (ii)

(i) – (ii)

$\frac{2}{3}S=1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+........$

$\frac{2}{3}S=\frac{4}{3}+\frac{5}{3^2}+\frac{5}{3^3}+..........$

$\frac{2}{3}S=\frac{4}{3}+\frac{\frac{5}{3^2}}{1-\frac{1}{3}}=\frac{4}{3}+\frac{5}{6}=\frac{13}{6}$

$S = \frac{13}{6}\times\frac{3}{2}=\frac{13}{4}$

8. The value of $\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$ is

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Ans. (1)

Sol. $= \lim_{x\to 0} \dfrac{4\left[\sin\left(\frac{\pi}{6}+x-\frac{\pi}{6}\right)\right]}{\sqrt{3}\,x(\sqrt{3})} = \lim_{x\to 0} \dfrac{4}{3}\dfrac{\sin x}{x} = \dfrac{4}{3}$

9. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\dfrac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where x $\in$ (0,1) is

(1) $\dfrac{10!}{\sqrt{3}(5!)^2}$ (2) $\dfrac{2.10!}{3\sqrt{3}(5!)^2}$ (3) $\dfrac{2.10!}{3(5!)^2}$ (4) $\dfrac{10!}{3(5!)^2}$

Ans. (2)

Sol. $T_{r+1} = {}^{10}C_r\,(t\,x^{1/5})^{10-r}\left(\dfrac{(1-x)^{1/10}}{t}\right)^r$

$10 - r - r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5\, x(1-x)^{1/2}$

$\dfrac{d(T_6)}{dx} = {}^{10}C_5\left((1-x)^{1/2} + \dfrac{-x}{2\sqrt{1-x}}\right) = 0$

$2(1-x) - x = 0 \Rightarrow x = \dfrac{2}{3}$

Maximum $T_6 = {}^{10}C_5 \dfrac{2}{3}\left(\dfrac{1}{3}\right)^{1/2} = 56\sqrt{3}$

10. The rate of growth of bacteria in a culture is proportional to the number of bacteris present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\dfrac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\dfrac{k}{\log_e 2}\right)^2$ is equal to

(1) 4 (2) 8 (3) 2 (4) 16

Ans. (1)

Sol. $\dfrac{dx}{dt} \propto x \qquad \Rightarrow \dfrac{dx}{dt} = \lambda x$

$\Rightarrow \displaystyle\int_{1000}^{x} \frac{dx}{x} = \lambda\int_0^t dt \qquad \Rightarrow \ell n\frac{x}{1000} = \lambda t$

at t = 2, x = 1200

$\therefore\ 2\lambda = \ell n\frac{6}{5}$

$\therefore\ x = 1000.e^{\frac{1}{2}\ell n\frac{6}{5}.t}$

Now $2000 = 1000.e^{\frac{1}{2}\ell n\frac{6}{5}.\frac{k}{\ell n\frac{5}{6}}} \Rightarrow 2 = e^{-\frac{k}{2}} \Rightarrow \frac{k}{2} = -\ell n2 \Rightarrow \frac{k}{\ell n2} = -2$

11. If (1,5,35), (7,5,5), (1,λ,7) and (2λ,1,2) are coplanar, then the sum of all possible values of λ is

(1) $\frac{39}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $-\frac{44}{5}$

Ans. (3)

Sol. for points to be coplanar $\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$

$\Rightarrow 6\ (-33\lambda + 165 - 112) + 30\ (2\lambda^2 - 11\lambda + 5) = 0$

$\Rightarrow 66\lambda^2 - 528\lambda - 522 = 0$

Sum $= \frac{528}{60} = \frac{44}{5}$

12. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}; 0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1 - y^2$ (3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

Ans. (3)

Sol. Let $\sin^{-1}x = a\lambda$, $\cos^{-1}x = b\lambda$, $\tan^{-1}y = c\lambda$

$\Rightarrow (a + b)\lambda = \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{a+b} = 2\lambda$

Now $\cos\left(\frac{\pi}{a+b}\right) = \cos(2\lambda c) = \cos\left(2\tan^{-1}y\right)$

$= \frac{1-y^2}{1+y^2}$

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 42 (2) 82 (3) 77 (4) 35

Ans. (3)

Sol. **Case-1:** 1, 1, 1, 1, 1, 2, 3

ways $= \frac{7!}{5!} = 42$

Case-2: 1, 1, 1, 1, 2, 2, 2

ways $= \frac{7!}{4!.3!} = 35$

total ways = 42 + 35 = 77

14. Let f be any function defined on R and let it satisfy the condition :

$|f(x) - f(y)| \le |(x - y)^2|, \forall (x,y) \in R$

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0, \forall x \in R$

(3) $f(x) = 0, \forall x \in R$

(4) $f(x) > 0, \forall x \in R$

Ans. (4)

Sol. $\left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$

$\Rightarrow |f'(x)| \le 0$

$\Rightarrow f'(x) = 0$

$\Rightarrow f(x) = \text{constant}$

$\Rightarrow f(x) = 1$

15. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point

(1) (2,2)

(2) (0,0)

(3) (2,9)

(4) $\left(3, \frac{21}{2}\right)$

Ans. (1)

Sol. $m = \frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36$

$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36 = 0$

$\Rightarrow x = 2, 3$

$\frac{d^2y}{dx^2} = 6(2x - 5)$

$\left.\frac{d^2y}{dx^2}\right|_{n=2} = -ve$

$\therefore$ Maximum at x = 2

Point (2, 46)

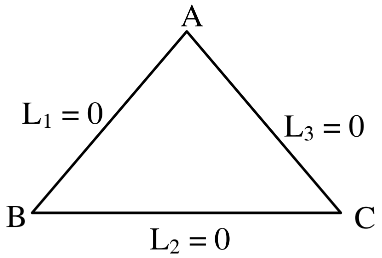
16. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

(1) Right angled triangle (2) Equilateral triangle

(3) Isosceles triangle (4) None of the above

Ans. (3)

Sol.



$L_1 : x - y = 0$
$L_2 : x + 2y = 3$
$L_3 : 2x + y = 6$
A (2, 2)
B (1, 1)
C (3, 0)
$\Rightarrow AB = \sqrt{2}$, $BC = \sqrt{5}$, $AC = \sqrt{5}$
$\therefore$ Triangle is isosceles

17. Consider the three planes
$P_1 : 3x + 15y + 21z = 9,$
$P_2 : x - 3y - z = 5,$ and
$P_3 : 2x + 10y + 14z = 5$
Then, which one of the following is true ?

(1) P_1 and P_2 are parallel (2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel (4) P_1, P_2 and P_3 all are parallel

Ans. (2)

Sol. $P_1 : x + 5y + 7z = 3$

$P_2 : x - 3y - z = 5$

$P_3 : x + 5y + 7z = 5/2$

P_1 and P_3 are parallel as dr's of normal are same

18. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) $(a + 2)(a + 3)(a + 4)$ (2) -2

(3) $(a + 1)(a + 2)(a + 3)$ (4) 0

Ans. (2)

Sol. $D = \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ a^2+5a+6 & a+3 & 1 \\ a^2+7a+12 & a+4 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$D = \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2a+4 & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix} = 4a + 8 - 4a - 10 = -2$

19. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is

(1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

Ans. (1)

Sol. $I = \int_0^{\pi/2} \left(\frac{\cos^2 x}{1+3^x} + \frac{\cos^2 x}{1+3^{-x}}\right) dx = \int_0^{\pi/2} \left(\frac{\cos^2 x}{1+3^x} + \frac{3^x \cos^2 x}{1+3^x}\right) dx = \int_0^{\pi/2} \cos^2 x \, dx$

$= \frac{1}{2}\int_0^{\pi/2} (1+\cos 2x)\, dx = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right)\Big|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$

20. Let R = {(P,Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set :

(1) $S = \{(x,y) \mid x^2 + y^2 = 4\}$ (2) $S = \{(x,y) \mid x^2 + y^2 = 1\}$

(3) $S = \{(x,y) \mid x^2 + y^2 = \sqrt{2}\}$ (4) $S = \{(x,y) \mid x^2 + y^2 = 2\}$

Ans. (4)

Sol. The equivalence class containing (1, –1) for this relation is $x^2 + y^2 = 2$

SECTION-B

1. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is

Ans. 2

Sol. order of differential equation is 1.

$2yy' = a$

$\Rightarrow y^2 = 2yy'\left(x+\sqrt{2yy'}\right)$

$\Rightarrow y - 2xy' = 2y'.\sqrt{2yy'}$

$\Rightarrow (y - 2xy')^2 = 4(y')^2.2yy'$

$\Rightarrow \left(y-2x.\frac{dy}{dx}\right)^2 = 8y.\left(\frac{dy}{dx}\right)^3$

Degree of Differential equation = 3

2. The number of integral values of 'k' for which the equation 3sinx + 4 cosx = k + 1 has a solution, $k \in R$ is

Ans. 11

Sol. $-\sqrt{3^2+4^2} \le 3 \sin x + 4 \cos x \le \sqrt{3^2+4^2}$

$-5 \le (k+1) \le 5$

$-6 \le k \le 4$

3. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is

Ans. 1

Sol. $\log_4(x-1) = \log_2(x-3)$

$\Rightarrow x - 1 = (x-3)^2 \quad \Rightarrow x = 2, 5$

but here $x \ge 3$

$\therefore\ x = 5$

4. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

Ans. 3

Sol. $x = 1, x = -\omega, x = -\omega^2$

$\alpha = 1, \beta = -\omega, \gamma = -\omega^2$

$E = 1 + \omega^{162} + (\omega^2)^{162}$

$= 3$

5. Let $m,n \in N$ and gcd (2,n) = 1. If $30\binom{30}{0}+29\binom{30}{1}+...+2\binom{30}{28}+1\binom{30}{29} = n.2^m$, then n + m is equal to

Ans. 45

Sol. General term = $(30-r).{}^{30}C_r$

L.H.S = $\sum_{r=0}^{30}(30-r).{}^{30}C_r$

$= 30\sum_{r=0}^{30}{}^{30}C_r - \sum_{r=0}^{30}r.{}^{30}C_r$

$= 30.2^{30} - 30.2^{29}$

$= 30.2^{29} = 15 \times 2^{30}$

So n = 15, m = 30

m + n = 45

6. If y = y(x) is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x, y(0) = 0$; then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to

Ans. 1

7. Let (λ,2,1) be a point on the plane which passes through the point (4,–2,2). If the plane is perpendicular to the line joining the points (–2,–21,29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to

Ans. 8

Sol. $\overrightarrow{AB} = \hat{i} + 5\hat{j} - 6\hat{k}$

$\vec{\alpha} = (\lambda - 4)\ \hat{i} + 4\hat{j} - \hat{k}$

$\overrightarrow{AB} \cdot \vec{\alpha} = 0$

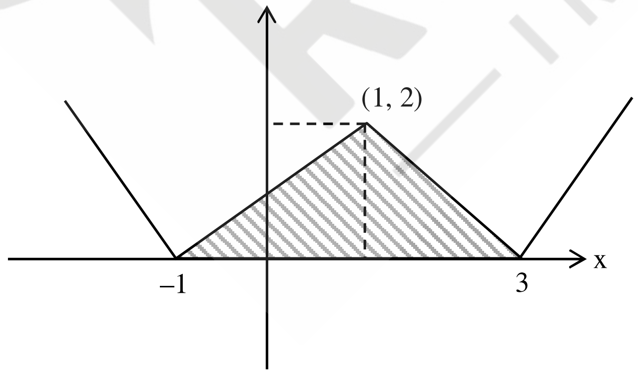
$\lambda - 4 + 20 + 6 = 0 \quad \Rightarrow \quad \lambda = -22$

$E = 4 + 8 - 4 = 8$

8. The area bounded by the lines $y = ||x - 1| - 2|$ is

Ans. 4

Sol.



$\text{Area} = \frac{1}{2} \times 4 \times 2 = 4$

Note : Tentatively this question is wrong because in the questions area is not bound. (x-axis is not given in the question)

9. The value of the integral $\int_0^{\pi}|\sin 2x|dx$ is

Ans. 2

Sol. $\int_0^{\pi}|\sin 2x|dx$

Here f(2a – x) = f(x)

$= 2\int_0^{\pi/2}(\sin 2x)dx$

$= 2\left(-\frac{\cos 2x}{2}\right)_0^{\pi/2}$

= 2

10. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

Ans. 1

Sol. $\sqrt{3}\cos^2 x - (\sqrt{3} - 1)\cos x - 1 = 0$

$\cos x = \frac{(\sqrt{3}-1) \pm \sqrt{(\sqrt{3}-1)^2 + 4\sqrt{3}}}{2\sqrt{3}}$

$= \frac{(\sqrt{3}-1) \pm \sqrt{(4+2\sqrt{3})}}{2\sqrt{3}} = \frac{(\sqrt{3}-1) \pm (\sqrt{3}+1)}{2\sqrt{3}}$

$= 1, \frac{-1}{\sqrt{3}}$

since $x \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos x = \frac{-1}{\sqrt{3}}$, not possible

$\therefore$ cos x = 1

$\Rightarrow$ x = 0

$\therefore$ number of solution 1