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PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

Date : 25 February, 2021 (SHIFT-2) Time ; (3.00 am to 6.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

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MATHEMATICS

SECTION-A

1. Let A be a 3 × 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \rightarrow 2R_2 + 5R_3$ on 2A, then det(B) is equal to :

(1) 16 (2) 80 (3) 128 (4) 64

Ans. (4)

Sol. $|A| = 4$

$|2A| = 2^3 |A| = 8 \times 4$

Now $R_2 \rightarrow 2R_2 + 5R_3$

$|B| = 2 \times 32 = 64$

2. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, x > 0, is equal to :

(where c is a constant of integration)

(1) $\log_e |x^2 + 5x - 7| + c$

(2) $4\log_e |x^2 + 5x - 7| + c$

(3) $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$

(4) $\log_e \sqrt{x^2 + 5x - 7} + c$

Ans. (2)

$I = \int \frac{8x^3 + 5(4x^2)}{x^4 + 5x^3 - 7x^2} dx \Rightarrow I = \int \frac{4(2x+5)}{x^2 + 5x - 7} dx$;

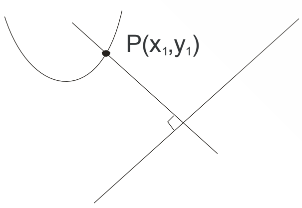
$\therefore \quad I = 4 \ell n |x^2 + 5x - 7| + C$

3. The shortest distance between the line x – y = 1 and the curve $x^2 = 2y$ is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) 0

Ans. (2)

Sol. $\left.\frac{dy}{dx}\right|_P = 1$



$\therefore x_1 = 1 \Rightarrow P = \left(1, \frac{1}{2}\right)$

$\therefore \; d_{min} = \left|\frac{1 - 1 - \frac{1}{2}}{\sqrt{2}}\right| = \frac{1}{2\sqrt{2}}$

4. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to :

(1) –3 (2) –7 (3) 7 (4) 3

Ans. (2)

Sol. If the root is $1 - 2i$, the other roots is $1 + 2i$

Sum = 2, Product = 5

$\therefore$ quadratic equation $z^2 - 2z + 5 = 0$

$\Rightarrow \alpha = -2, \beta = 5$

$\alpha - \beta = -2 - 5 = -7$

5. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities in one, then the equation of the hyperbola is :

(1) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Ans. (2)

Sol. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$a = 5, b = 4$

$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

focii : (3, 0), (–3, 0)

let equation of hyperbola is $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

satisfy $(\pm 3, 0) \Rightarrow \frac{9}{A^2} = 1 \Rightarrow A^2 = 9$

eccentricity of hyperbola $= \frac{1}{\text{eccentricity of ellipse}} = \frac{5}{3}$

$\Rightarrow \frac{5}{3} = \sqrt{1 + \frac{B^2}{9}} \Rightarrow 1 + \frac{B^2}{9} = \frac{25}{9}$

$\Rightarrow B^2 = 16$

equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

6. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1+\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

Ans. (2)

Sol. $x = y = \frac{\pi}{3}$ satisfy the equation

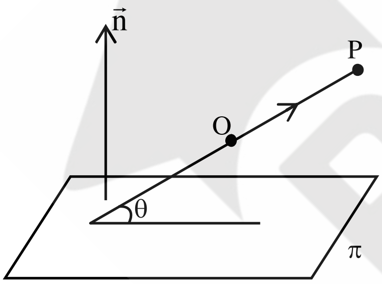
$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

7. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length :

(1) $\sqrt{\frac{2}{7}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{5}}$

Ans. (3)

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 0 & 1 & 1 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$



$\therefore$ Required plane is : $3(x - 2) - (y - 1) + (z - 3) = 0$

i.e. $3x - y + z = 8$

$\overrightarrow{OP} = 2\hat{i} - \hat{j} + \hat{k}$

$|\overrightarrow{OP}| = \sqrt{4+1+1} = \sqrt{6}$

$\sin\theta = \left|\frac{6+1+1}{\sqrt{6}\sqrt{11}}\right| = \frac{8}{\sqrt{66}}$

$\therefore$ Projection $= \sqrt{6} \times \cos\theta = \sqrt{6} \times \sqrt{\frac{2}{66}} = \sqrt{\frac{2}{11}}$

8. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is :

(1) $\frac{7}{45}$ (2) $\frac{14}{45}$ (3) $\frac{28}{45}$ (4) $\frac{8}{45}$

Ans. (3)

Sol. Nonveg + smoker $\frac{160}{400}\xrightarrow{\text{disease}}\frac{160}{400}\times\frac{35}{100}$

Veg + smoker $\frac{100}{400}\xrightarrow{\text{disease}}\frac{100}{400}\times\frac{20}{100}$

Veg + smoker $\frac{140}{400}\xrightarrow{\text{disease}}\frac{140}{400}\times\frac{10}{100}$

$$\text{Required probability} = \frac{\frac{160\times35}{400\times100}}{\frac{160\times35}{400\times100}+\frac{100\times20}{400\times100}+\frac{140\times10}{400\times100}}$$

$$= \frac{16\times35}{16\times35+10\times20+140}=\frac{560}{900}=\frac{56}{90}=\frac{28}{45}$$

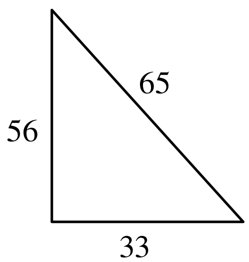
9. $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to :

(1) $\frac{56}{33}$ (2) $\frac{65}{56}$ (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

Ans. (2)

Sol. $= \operatorname{cosec}\left(\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right)$

$= \operatorname{cosec}\left(\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right)$



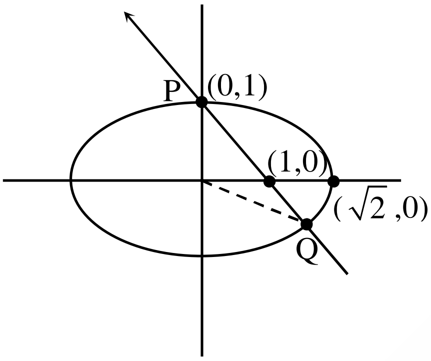
$= \operatorname{cosec}\left(\tan^{-1}\left(\frac{56}{33}\right)\right) = \frac{65}{56}$

10. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is :

(1) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$ (2) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)$ (3) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$ (4) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

Ans. (4)

Sol.



Homogenise Ellipse w.r.t. line, $\frac{x^2}{2}+\frac{y^2}{1}=(x+y)^2$

$\therefore\ x^2 + 2y^2 = 2x^2 + 2y^2 + 4xy$

$\Rightarrow x^2 + 4xy = 0$ $\Rightarrow x = 0\ ,\ y = -\frac{x}{4}$

angle between these line is $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

11. The contrapositive of the statement "If you will work, you will earn money" is :

(1) You will earn money, if you will not work

(2) If you will earn money, you will work

(3) If you will not earn money, you will not work

(4) To earn money, you need to work

Ans. (3)

Sol. p : you will work

q : you will earn money

given $(p \to q)$

contrapositive of $(p \to q) = \sim q \to \sim p$

12. A function f(x) is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series

$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+......+f\left(\frac{39}{20}\right)$ is equal to :

(1) $\frac{19}{2}$ (2) $\frac{49}{2}$ (3) $\frac{29}{2}$ (4) $\frac{39}{2}$

Ans. (4)

Sol. $f(x)=\frac{5^x}{5+5^x}$

$f(2-x)=\frac{5^{2-x}}{5+5^{2-x}}$

$=\frac{25}{5.5^x+25}=\frac{5}{5^x+5}$

$f(x)+f(2-x)=1$

Now $\left(f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)\right)+\left(f\left(\frac{2}{20}\right)+f\left(\frac{38}{20}\right)\right)+\ldots\ldots\left(f\left(\frac{19}{12}\right)+f\left(\frac{21}{20}\right)\right)+f\left(\frac{20}{20}\right)$

$=1\times 19+\frac{1}{2}=\frac{39}{2}$

13. If for the matrix, $A=\begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}$, $AA^T=I_2$, then the value of $\alpha^4+\beta^4$ is :

(1) 4 (2) 2 (3) 3 (4) 1

Ans. (4)

Sol. $AA^T=I$

$\Rightarrow \begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}\begin{bmatrix}1 & \alpha\\ -\alpha & \beta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1+\alpha^2 & \alpha-\alpha\beta\\ \alpha-\alpha\beta & \alpha^2+\beta^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow 1+\alpha^2=1\Rightarrow \alpha=0$

$\alpha^2+\beta^2=1\Rightarrow \beta^2=1$

$\therefore\ \alpha^4+\beta^4=0+1=1$

14. The minimum value of $f(x)=a^{a^x}+a^{1-a^x}$, , where a, x $\in$ R and a > 0, is equal to :

(1) 2a (2) $2\sqrt{a}$ (3) $a+\frac{1}{a}$ (4) a + 1

Ans. (2)

Sol. $A.M\geq G.M\Rightarrow \frac{a^{a^x}+\frac{a}{a^{a^x}}}{2}\geq\left(a^{a^x}\times\frac{a}{a^{a^x}}\right)^{1/2}$

$a^{a^x}+\frac{a}{a^{a^x}}\geq 2\sqrt{a}$

$\therefore$ Minimum value $=2\sqrt{a}$

15. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x\, dx$, then :

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(2) $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in A.P.

(3) $I_2 + I_4$, $(I_3 + I_5)^2$, $I_4 + I_6$ are in G.P.

(4) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

Ans. (4)

Sol. $I_n = \int_{\pi/4}^{\pi/2} (\cot x)^n\, dx$

$$I_n + I_{n+2} = \int_{\pi/4}^{\pi/2} ((\cot x)^n + (\cot x)^{n+2})dx = \int_{\pi/4}^{\pi/2} (\cot x)^n \operatorname{cosec}^2 x\, dx$$

$\cot x = t$

$$= -\int_{1}^{0} t^n dx = \int_{0}^{1} t^n dx = \left.\frac{t^{n+1}}{n+1}\right|_0^1 = \frac{1}{n+1}$$

$\therefore \quad I_n + I_{n+2} = \frac{1}{n+1}$

$\therefore \quad I_2 + I_4 = \frac{1}{3}$

$I_3 + I_5 = \frac{1}{4}$

$I_4 + I_6 = \frac{1}{5}$

$\Rightarrow \quad I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in H.P.

16. $\lim_{n\to\infty}\left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \ldots\ldots + \frac{n}{(2n-1)^2}\right]$ is equal to :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Ans. (1)

Sol. $1 = \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}$

$\therefore L = \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\left(\frac{r}{n}\right)^2+2\left(\frac{r}{n}\right)+1}$

$\therefore L = \int_0^1\frac{dx}{(x+1)^2} = \left.\frac{-1}{x+1}\right|_0^1$

$L = -\frac{1}{2}+1 = \frac{1}{2}$

17. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

(1) $\frac{2}{9}$ (2) $\frac{122}{297}$ (3) $\frac{97}{297}$ (4) $\frac{1}{5}$

Ans. (3)

Sol. $n(A) = 7......... + \underbrace{..................}_{7}$

$= 1\times9\times9\times9+8\times{}^3C_1\times1\times9\times9$

$= 729 + 1944 = 2673$

Favourable : ....7 + $\underbrace{................2}_{\text{7 exactly once}}$

$= 8\times9\times9+1\times9\times9\times1+2\times8\times1\times9$

$= 648 + 81 + 144$

$= 873$

$\therefore$ Probability $= \frac{873}{2673} = \frac{97}{297}$

18. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is :

(1) 2 (2) 1 (3) 4 (4) 3

Ans. (1)

Sol. $E = \frac{(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)}{3(\alpha^9-\beta^9)}$

$= \frac{\alpha^8(\alpha^2-2)-\beta^8(\beta^2-2)}{3(\alpha^9-\beta^9)}$

$= \frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{3(\alpha^9-\beta^9)} = \frac{6(\alpha^9-\beta^9)}{3(\alpha^9-\beta^9)} = 2$

19. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then :

(1) y = 273x (2) 2y = 91x (3) y = 91x (4) 2y = 273x

Ans. (2)

$x = {}^5C_3 \times 3! = 60$

$y = {}^{15}C_3 \times 3! = 2730$

$\therefore \quad 2y = 91x$

20. The following system of linear equations

$2x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(2) has infinitely many solutions

(3) does not have any solution

(4) has a unique solution

Ans. (4)

Sol. $D = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = -20 \neq 0$

so unique solution

SECTION-B

1. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ______.

Ans. 45

Sol. n is odd number

Hence n = {11, 13, 15, ….., 99}

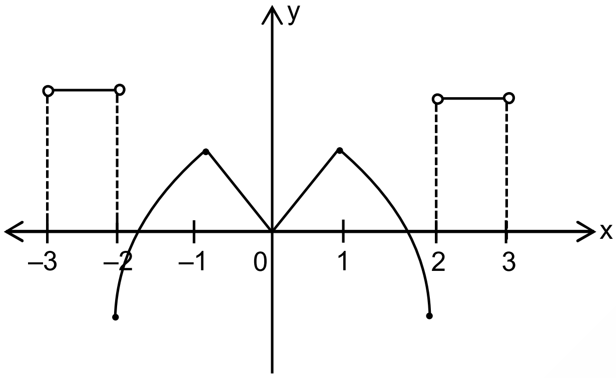
Number of values of ‘n’ is 45

2. A function f is defined on [–3, 3] as $f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \le x \le 2 \\ [|x|] & , 2 < |x| \le 3 \end{cases}$

where [x] denotes the greatest integer ≤ x. The number of points, where f is not differentiable in (3, 3) is ______.

Ans. 5

Sol. Using graph of f(x)



5 point

3. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to ______:

Ans. 2

Sol. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix} = 4\alpha\hat{i} - 8\hat{j} - 4\alpha\hat{k}$

area = $\left|\bar{a}\times\bar{b}\right| = 8\sqrt{3}$

= $\sqrt{16\alpha^2 + 16\alpha^2 + 64} = 8\sqrt{3}$

$32\alpha^2 + 64 = 64.3$

$\alpha^2 + 2 = 2.3 = 6 \Rightarrow a^2 = 4$

$\alpha = \pm 2$

$\bar{a}.\bar{b} = 3 - \alpha^2 + 3 = 6 - 4 = 2$

4. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _______.

Ans. 1

Sol. $x = 4k + 3$; $k \in W$

$\therefore (2020 + 4k + 3)^{2022} = (8\lambda + 1)^{1011}$

$\therefore (8\lambda + 1)^{1011} = {}^{1011}C_0 + \underbrace{{}^{1011}C_1(8\lambda) + \ldots\ldots\ldots}_{\text{multiple of 8.}}$

$\therefore$ Remainder on dividing by 8 is 1

5. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to ______.

Ans. 4

Sol. Point of intersection (P) $\equiv (k^{4/5}, k^{1/5})$

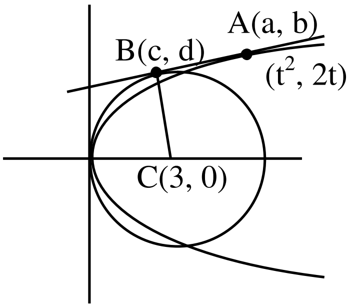
$\therefore \quad \dfrac{1}{4k^{3/5}} \times (-k^{-3/5}) = -1$

$\Rightarrow \quad k^{6/5} = \dfrac{1}{4} \qquad\qquad \Rightarrow \quad (4k)^6 = 4$

6. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a + c) is equal to ______.

Ans. 9

Sol. Equation of tangent of A



$ty = x + t^2$

$x - yt + t^2 = 0$

$\left|\frac{3-0+t^2}{\sqrt{1+t^2}}\right| = 3$

$(3 + t^2)^2 = 9(1 + t^2)$

$t = 0, \pm\sqrt{3}$

Point A $(3, 2\sqrt{3})$ in first quadrant

For point B foot of perpendicular from c to tangent

$\frac{x-3}{1} = \frac{y-0}{-\sqrt{3}} = -\frac{(3-0+3)}{4} \Rightarrow x = \frac{3}{2}$

$c = \frac{3}{2}$ and $a = 3$

$2(a + c) = 9$

7. If $\lim_{x\to 0}\frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of a – 2b is ______.

Ans. 5

Sol. $\lim_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax.4x}$

Apply L' Hospital

$\lim_{x\to 0}\frac{a-\left(e^{4x}\right).4}{8ax}$ $\qquad$ $\left(\frac{a-4}{0}\text{ form}\right)$

4 limit exist a = 4

$\lim_{x\to 0}\frac{4-4e^{4x}}{32x} = \lim_{x\to 0}\frac{1-e^{4x}}{8x} = \frac{-1}{2}$

$a = 4,\ b = \frac{-1}{2}$

$2(a + b) = 2\left(4-\frac{1}{2}\right) = 7$

8. If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to ______.

Ans. 1

Sol. $(2xy^2 - y)\,dx = -x\,dy$

$$x\frac{dy}{dx} = y - 2xy^2$$

$$\frac{dy}{dx} = \frac{1}{x}\,.\,y - 2xy^2$$

$$\frac{dy}{dx} - \frac{1}{x}\,.\,y = -2xy^2$$

$$y^{-2}\frac{dy}{dx} - \frac{1}{x}\,y^{-1} = -2$$

$$y^{-1} = t \Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dt}{dx}$$

$$-\frac{dt}{dx} - \frac{1}{x}\,t = -2$$

$$\Rightarrow \frac{dt}{dx} + \frac{1}{x}\,.\,t = 2 \qquad \text{I.F.} = e^{\ell nx} = x$$

$$tx = 2\times\frac{x^2}{2} + c \qquad \Rightarrow \frac{x}{y} = x^2 + c$$

It passes through P(2, 1)

$\therefore\ c = -2$

$\therefore\ \frac{x}{y} = x^2 - 2$

$\therefore\ |y(1)| = 1$

9. The value of $\int_{-2}^{2}\left|3x^2 - 3x - 6\right| dx$ is ______.

Ans. 19

Sol. $\frac{I}{3} = \int_0^2 |(x-2)(x+1)| + |(-x-2)(-x+1)|\,dx$

$$= \int_0^2 |((-x^2 + x + 2) + (x+2)|x-1|)\,dx$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_0^2 + \int_0^1 (x+2)(-x+1)\,dx + \int_1^2 (x^2 + x - 2)\,dx$$

$$= \left(-\frac{8}{3}+2+4\right) + \int_0^1 (-x^2 - x + 2)\,dx + \left(\frac{x^3}{3}+\frac{x^2}{2}-2x\right)\Bigg|_1^2$$

$$= \frac{10}{3} + \left(-\frac{x^3}{3}-\frac{x^2}{2}+2x\right)\Bigg|_0^1 + \left(\frac{8}{3}+2-4\right) - \left(\frac{1}{3}+\frac{1}{2}-2\right)$$

$$= \frac{10}{3} + \left(-\frac{1}{3}-\frac{1}{2}+2\right) + \frac{2}{3} + \frac{7}{6} = \frac{19}{3}$$

$\therefore$ I = 19

10. A line 'l' passing through origin is perpendicular to the lines

$l_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

If the co-ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c), then 18(a + b + c) is equal to ______.

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$\ell_1 : \vec{r} = (3\hat{i} - \hat{j} + 4\hat{k}) + t\,(\hat{i} + 2\hat{j} + 2\hat{k})$

$\ell_2 : \vec{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + s\,(2\hat{i} + 2\hat{j} + \hat{k})$

$$\vec{n} : \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

Point on ℓ_2 : (3 + 2s, 3 + 2s, 2 + s)

$\ell = \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} \rightarrow (2\lambda, -3\lambda, 2\lambda)$

$\therefore$ Point of intersection of ℓ & $\ell_1 \equiv (2, -3, 2)$

$\therefore$ $(1 + 2s)^2 + (6 + 2s)^2 + s^2 = 17$

$\Rightarrow$ $s = -\frac{10}{9}$

$\therefore$ $(a, b, c) \equiv \left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$