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# PAPER-1(B.E./B. TECH.)

# JEE (Main) 2021

## Questions & Solutions

**Date : 24 February, 2021 (SHIFT-1) Time ; (9.00 am to 12.00 pm)**

**Duration : 3 Hours | Max. Marks : 300**

**SUBJECT : MATHEMATICS**

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# MATHEMATICS

## SECTION-A

**1.** The statement among the following that is a tautology is :

(1) $A \vee (A \wedge B)$  (2) $A \wedge (A \vee B)$

(3) $B \to [A \wedge (A \to B)]$  (4) $[A \wedge (A \to B)] \to B$

**Ans. (4)**

**Sol.** $[A \wedge (\sim A \vee B)] \to B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \to B$

$= (A \wedge B) \to B$

$= \sim A \vee \sim B \vee B$

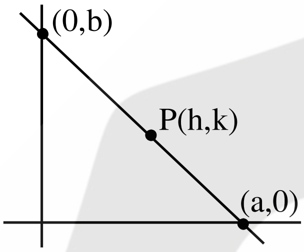
$= t$

**2.** A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1,1), (2, 2) and (4, 4) respectively. Then which of these stones is / are on the path of the man ?

(1) A only  (2) C only  (3) All the three  (4) B only

**Ans. (4)**

**Sol.**




$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{h}{a} + \frac{k}{b} = 1$ ........ (i)

$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$

$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2}$ ........ (ii)

$\therefore$ Line passes through fixed point (2, 2)

(from (1) and (2))

**3.** The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes 3x + y – 2z = 5 and 2x – 5y – z = 7, is

(1) $3x - 10y - 2z + 11 = 0$  (2) $6x - 5y - 2z - 2 = 0$

(3) $11x + y + 17z + 38 = 0$  (4) $6x - 5y + 2z + 10 = 0$

**Ans. (3)**

**Sol.** Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore$ +11 (x – 1) + (y – 2) + 17 (z + 3) = 0

11x + y + 17z + 38 = 0

**4.** The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If P(0) = 850, then the time at which population becomes zero is :

(1) $\log_e 18$ (2) $\log_e 9$ (3) $\frac{1}{2}\log_e 18$ (4) $2\log_e 18$

**Ans.** **(4)**

**Sol.** $\frac{dP(t)}{dt} = \frac{P(t)-900}{2}$

$\int_0^t \frac{dP(t)}{P(t)-900} = \int_0^t \frac{dt}{2}$

$\{\ell n\,|P(t)-900|\}_0^t = \left\{\frac{t}{2}\right\}_0^t$

$\ell n|P(t) - 900| - \ell n|P(0) - 900| = \frac{t}{2}$

$\ell n|P(t) - 900| - \ell n 50 = \frac{t}{2}$

Let at t = $t_1$ , P(t) = 0 hence

$\ell n|P(t) - 900| - \ell n\ 50 = \frac{t_1}{2}$

$t_1 = 2\ell n 18$

**5.** The system of linear equations

3x – 2y – kz = 10

2x – 4y – 2z = 6

x + 2y – z = 5m

is inconsistent if :

(1) $k = 3,\ m = \frac{4}{5}$ (2) $k \neq 3,\ m \in R$ (3) $k \neq 3,\ m \neq \frac{4}{5}$ (4) $k = 3,\ m \neq \frac{4}{5}$

**Ans.** **(4)**

**Sol.** $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0 \Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} = 8(4-5m)$

$\Delta_x \neq 0 \quad \Rightarrow \quad m \neq \frac{4}{5}$

Hence $k = 3,\ m \neq \frac{4}{5}$

**6.** If $f : R \to R$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then $f$ is :

(1) discontinuous at all integral values of x except at x = 1
(2) continuous only at x = 1
(3) continuous for every real x
(4) discontinuous only at x = 1

**Ans.** **(3)**

**Sol.** Doubtful points are $x = n,\ n \in I$

L.H.L $= \lim_{x \to n^-} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$

R.H.L. $= \lim_{x \to n^+} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$

$f(n) = 0$

Hence continuous

**7.** The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is :

(1) $2\sqrt{19}$ (2) $19\sqrt{2}$ (3) 38 (4) $\sqrt{38}$

**Ans.** **(4)**

**Sol.** $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$

$\Rightarrow x = \lambda + 3,\ y = 2\lambda + 4,\ z = 2\lambda + 5$

Which lies on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda = \frac{5}{5} = 1$

Hence , point of intersection is Q (4,6,7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

**8.** If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) 0 (3) $-t^3$ (4) $2t^3$

**Ans.** **(1)**

**Sol.** equation of tangent at $P(t,t^3)$

$(y - t^3) = 3t^2 (x - t)$ .......(1)

now solve the above equation with

$y = x^3$ .......(2)

By (1) & (2)

$x^3 - t^3 = 3t^2 (x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point $= \dfrac{2t^3 + (-8t^3)}{3} = -2t^3$

**9.** If $\int \dfrac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\dfrac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (–1, 3) (2) (3, 1) (3) (1, 3) (4) (1, –3)

**Ans.** **(3)**

**Sol.** put $\sin x + \cos x = t \Rightarrow 1 + \sin 2x = t^2$

$\Rightarrow (\cos x - \sin x)\, dx = dt$

$\therefore I = \int \dfrac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \dfrac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\dfrac{t}{3}\right) + C = \sin^{-1}\left(\dfrac{\sin x + \cos x}{3}\right) + C$

$\Rightarrow$ a = 1 and b = 3

**10.** The value of $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ......$

$-15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + .... + {}^{14}C_{11}$ is :

(1) $2^{16} - 1$ (2) $2^{13} - 14$ (3) $2^{14}$ (4) $2^{13} - 13$

**Ans.** **(2)**

**Sol.** $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - .........- 15.{}^{15}C_{15}$

$= \sum_{r=1}^{15} (-1)^r . r . {}^{15}C_r = 15 \sum_{r=1}^{15} (-1)^r \; {}^{14}C_{r-1}$

$= 15(-{}^{14}C_0 + {}^{14}C_1 - .....- {}^{14}C_{14}) = 15(0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + .........+ {}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 + ........+ {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

$S_1 + S_2 = 2^{13} - 14$

**11.** The function $f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$ :

(1) increases in $\left[\frac{1}{2},\infty\right)$ (2) increases in $\left(-\infty,\frac{1}{2}\right]$

(3) decreases in $\left[\frac{1}{2},\infty\right)$ (4) decreases in $\left(-\infty,\frac{1}{2}\right]$

**Ans.** **(1)**

**Sol.** $f'(x) = (2x-1)(x-\sin x)$

$\Rightarrow f'(x) \geq 0$ in $x \in \left[\frac{1}{2},\infty\right)$

and $f'(x) \leq 0$ in $x \in \left(-\infty,\frac{1}{2}\right]$

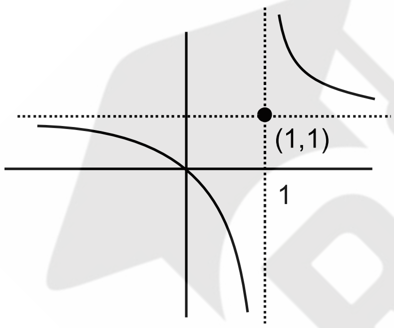
**12.** Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x)=\frac{x-\frac{1}{2}}{x-1}$. Then the composition function $f(g(x))$ is :

(1) onto but not one-one (2) both one-one and onto

(3) one-one but not onto (4) neither one-one nor onto

**Ans.** **(3)**

**Sol.**




$f(g(x)) = 2g(x) - 1$

$= 2\frac{\left(x-\frac{1}{2}\right)}{x-1} - 1 = \frac{x}{x-1}$

$f(g(x)) = 1 + \frac{1}{x-1}$

one-one, into

**13.** An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{1}{32}$ (2) $\frac{5}{16}$ (3) $\frac{3}{16}$ (4) $\frac{1}{2}$

**Ans.** **(4)**

**Sol.** P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$

success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

**14.** A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

(1) 1625 (2) 575 (3) 560 (4) 1050

**Ans.** **(1)**

**Sol.** (2I , 4F) + (3I , 6F) + (4I , 8F)

$= {}^6C_2\,{}^8C_4 + {}^6C_3\,{}^8C_6 + {}^6C_4\,{}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

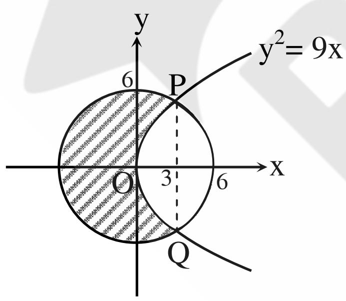
$= 1050 + 560 + 15 = 1625$

**15.** The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :

(1) $24\pi + 3\sqrt{3}$ (2) $12\pi - 3\sqrt{3}$ (3) $24\pi - 3\sqrt{3}$ (4) $12\pi + 3\sqrt{3}$

**Ans.** **(3)**

**Sol.** The curves intersect at points $(3, \pm 3\sqrt{3})$




Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left|\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right|_3^6$$

$$= 36p - 12\sqrt{3} - 2\left(9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$

**16.** Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$　　(2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$　　(4) $x^2 - 2x + 16 = 0$

**Ans.** **(4)**

**Sol.** $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 - 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

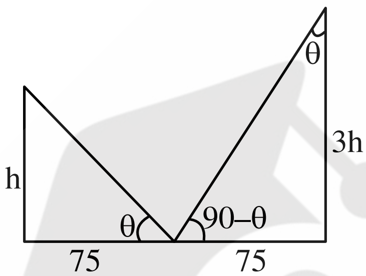
$pq = 16$

**17.** Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

(1) $20\sqrt{3}$　　(2) $25\sqrt{3}$　　(3) 30　　(4) 25

**Ans.** **(2)**

**Sol.**




$\tan\theta = \frac{h}{75} = \frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}$ m

**18.** $\lim_{x\to 0} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$　　(2) $\frac{3}{2}$　　(3) 0　　(4) $\frac{1}{15}$

**Ans.** **(1)**

**Sol.** $\lim_{x\to 0^+} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3} = \lim_{x\to 0^+} \frac{(\sin|x|)2x}{3x^2} = \lim_{x\to 0^+} \left(\frac{\sin x}{x}\right) \times \frac{2}{3} = \frac{2}{3}$

**19.** If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+.....\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is

(1) $2\sqrt{3}$ (2) $\frac{3}{2}$ (3) $\sqrt{3}$ (4) $\frac{1}{2}$

**Ans.** **(4)**

**Sol.** $e^{(\cos^2\theta+\cos^4\theta+....\infty)\ln 2} = 2^{\cos^2\theta+\cos^4\theta+....\infty}$

$= 2^{\cot^2\theta}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2\theta} = 1, 8 \Rightarrow \cot^2\theta = 0, 3$

$0<\theta<\frac{\pi}{2} \Rightarrow \cot\theta = \sqrt{3}$

$\Rightarrow \frac{2\sin\theta}{\sin\theta+\sqrt{3}\sin\theta} = \frac{2}{1+\sqrt{3}\cot\theta} = \frac{2}{4} = \frac{1}{2}$

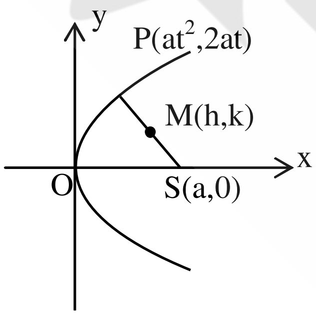
**20.** The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :

(1) $x = -\frac{a}{2}$ (2) $x = \frac{a}{2}$ (3) $x = 0$ (4) $x = a$

**Ans.** **(3)**

**Sol.** $h = \frac{at^2+a}{2}$, $k = \frac{2at+0}{2}$

$\Rightarrow t^2 = \frac{2h-a}{a}$ and $t = \frac{k}{a}$




$\Rightarrow \frac{k^2}{a^2} = \frac{2h-a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x-\frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

## SECTION-B

**1.** If the least and the largest real values of $\alpha$, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ____

**Ans.** **(10)**

**Sol.** $x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$\therefore\ y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2+y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$$\alpha^2 = \frac{x^2}{x^2 - 2x + 5}$$

$$\alpha^2 \in \left[0, \frac{5}{4}\right]$$

$$\therefore\ \alpha^2 \in \left[0, \frac{5}{4}\right]$$

$$\therefore\ \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

then $4[(\alpha_{max})^2 + (\alpha_{min})^2] = 4\left[\frac{5}{4} + \frac{5}{4}\right] = 10$

**2.** If $\int_{-a}^{a}(|x| + |x-2|)dx = 22,\ (a > 2)$ and [x] denotes the greatest integer $\le x$, then $\int_{a}^{-a}(x + [x])\,dx$ is equal to _______.

**Ans.** **(3)**

**Sol.** $\int_{-a}^{0}(-2x+2)\,dx + \int_{0}^{2}(x+2-x)\,dx + \int_{2}^{a}(2x-2)\,dx = 22$

$$x^2 - 2x\Big|_0^{-a} + 2x\Big|_0^2 + x^2 - 2x\Big|_2^a = 22$$

$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$$\int_{-3}^{3}(x + [x])\,dx = -3 - 2 - 1 + 1 + 2 = -3$$

**3.** Let $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

and $C = \{9k + l : k \in N\}$ for some $l$ $(0 < l < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is $274 \times 400$, then $l$ is equal to ______.

**Ans.** **(5)**

**Sol.** 3 digit number of the form 9K + 2 are {101,109,..............,992}

$\Rightarrow$ Sum equal to $\frac{100}{2}$ (1093)

Similarly sum of 3 digit number of the form 9K + 5 is $\frac{100}{2}$ (1099)

$\frac{100}{2}(1093) + \frac{100}{2}(1099) = 100 \times (1096)$

$= 400 \times 274$

$\Rightarrow \ell = 5$

**4.** Let M be any 3 × 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of $M^TM$ is seven, is _______.

**Ans.** **(540)**

**Sol.** $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

${}^9C_2 = 36$

Case- II : One (2) and three (1's) and five (0's)

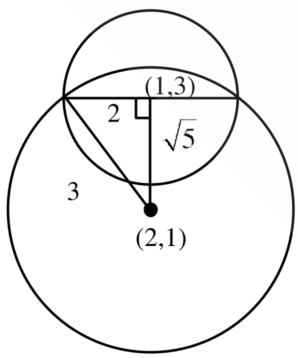
$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540

**5.** If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is ______ .

**Ans.** **(3)**

**Sol.**




distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

**6.** The minimum value of $\alpha$ for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=\alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _______.

**Ans.** **(9)**

**Sol.** Let $f(x) = \frac{4}{\sin x}+\frac{1}{1-\sin x}$

$$y = \frac{4-3\sin x}{\sin x(1-\sin x)}$$

Let $\sin x = t$ when $t \in (0,1)$

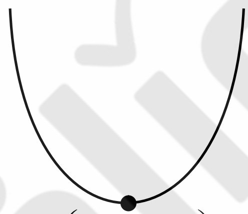
$$y = \frac{4-3t}{t-t^2}$$

$$\frac{dy}{dt}=\frac{-3(t-t^2)-(1-2t)(4-3t)}{(t-t^2)^2}=0$$

$\Rightarrow 3t^2 - 3t - (4 - 11t + 6t^2) = 0$

$\Rightarrow 3t^2 - 8t + 4 = 0$

$\Rightarrow 3t^2 - 6t - 2t + 4 = 0$



$\left(\sin^{-1}\frac{2}{3}, 9\right)$

$\Rightarrow t = \frac{2}{3}$

$\Rightarrow \alpha \geq 9$

least $\alpha$ is equal to 9

**7.** $\lim\limits_{n\to\infty} \tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ____.

**Ans.** **(1)**

**Sol.** $\tan\left(\lim\limits_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$

$= \tan\left(\lim\limits_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

**8.** Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is ______.

**Ans.** **(75)**

**Sol.** $\vec{c} = \lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$= \lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b} - \left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$= \lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$

$= \lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$

$\lambda = \frac{1}{2}$

$\therefore 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2 = 2\left(\frac{1}{4}+\frac{49}{4}+25\right)$ = 25 + 50 = 75

**9.** Let $B_i$ ($i$ = 1, 2, 3) be three independent events in a sample space. The probability that only $B_1$ occur is $\alpha$, only $B_2$ occurs is $\beta$ and only $B_3$ occurs is $\gamma$. Let p be the probability that none of the events $B_i$ occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0,1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to______.

**Ans.** **(6)**

**Sol.** Let x, y, z be probability of $B_1$, $B_2$, $B_3$ respectively

$\Rightarrow$ $x(1-y)(1-z) = \alpha \Rightarrow$ $y(1-x)(1-z) = \beta \Rightarrow$ $z(1-x)(1-y) = \gamma \Rightarrow (1-x)(1-y)(1-z) = P$

Putting in the given relation we get x = 2y and y = 3z $\Rightarrow$ x = 6z $\Rightarrow$ $\frac{x}{z} = 6$

**10.** Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _______.

**Ans.** **(17)**

**Sol.** As PQ = kI $\Rightarrow$ $Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(\text{adj}P)\,I \Rightarrow$ $Q = \frac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha-4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8}$ $\Rightarrow$ $\frac{k}{(20+12\alpha)}(-3\alpha-4) = \frac{-k}{8}$ $\Rightarrow$ $2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3 \Rightarrow \alpha = -1$

also $|Q| = \frac{k^3\,|I|}{|P|}$ $\Rightarrow$ $\frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20 + 12\alpha) = 2k$ $\Rightarrow$ $8 = 2k$ $\Rightarrow$ $k = 4$