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PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

Date : 24 February, 2021 (SHIFT-2) Time ; (3.00 pm to 6.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

A-10 Road No. 1, IPIA, Kota-324005 (Rajasthan), India
Tel. : + 91-744-2665544 | Website : www.reliablekota.com | E-mail: info@reliablekota.com

MATHEMATICS

SECTION-A

1. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.
(2) (a) and (b) both are tautologies.
(3) (a) is a tautology but not (b).
(4) (b) is a tautology but not (a).

Ans. **(2)**

Sol. Truth table

p	q	~ p	~ q	p∨q	p→q	~q∧(p→q)	(p∨q)∧ ~p	(a)	(b)
T	T	F	F	T	T	F	F	T	T
T	F	F	T	T	F	F	F	T	T
F	T	T	F	T	T	F	T	T	T
F	F	T	T	F	T	T	F	T	T

2. Let $a, b \in R$. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, –a –9), then |a + b| is equal to :

(1) 88
(2) 86
(3) 84
(4) 90

Ans. **(1)**

Sol. mid point of p(a,6,9) and (20,b,–a–9) is $\left(\frac{a+20}{2},\frac{6+b}{2},\frac{-a}{2}\right)$ lies on the line

So $\frac{\frac{a+20}{2}-3}{7}=\frac{\frac{6+b}{2}-2}{5}=\frac{\frac{-a}{2}-1}{-9}$

$\frac{a+14}{14}=\frac{2+b}{10}=\frac{a+2}{18} \Rightarrow b=-32, a=-56$

$\Rightarrow |a+b| = 88$

3. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is :

(1) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$
(2) $\vec{r}.(3\hat{i}+7\hat{j}+3\hat{k})=7$
(3) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=7$
(4) $\vec{r}.(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

Ans. **(3)**

Sol. Plane passing through intersection of plane is

$\{\vec{r}\,.(\hat{i}+\hat{j}+\hat{k})=-1\} + \lambda\{\vec{r}\,.(\hat{i}-2\hat{j})+2\}=0$

Passes through $\hat{i}+2\hat{k}$, we get

$(3-1)+\lambda(\lambda+2)=0 \quad \Rightarrow \quad \lambda=-\frac{2}{3}$

Hence, equation of plane is $\quad 3\{\vec{r}\,.(\hat{i}+\hat{j}+\hat{k})-1\} - 2\{\vec{r}\,.(\hat{i}-2\hat{j})+2\} = 0$

$\Rightarrow \quad \vec{r}\,.(\hat{i}+7\hat{j}+3\hat{k})=7$

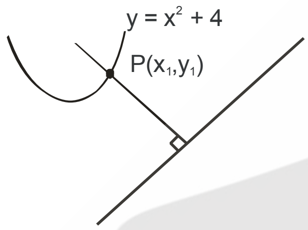
4. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(1) (3, 13) (2) (1, 5) (3) (–2, 8) (4) (2, 8)

Ans. **(4)**

Sol. $\left.\frac{dy}{dx}\right|_p = 4$

$\therefore\ 2x_1 = 4$



$\Rightarrow x_1 = 2$

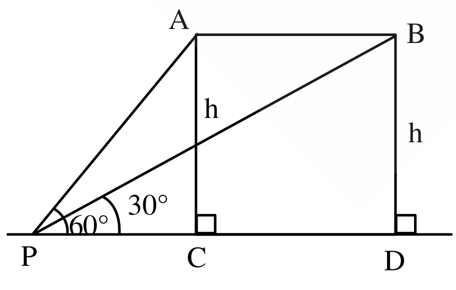
$\therefore$ Point will be (2,8)

5. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

(1) $1800\sqrt{3}$ m (2) $3600\sqrt{3}$ m (3) $2400\sqrt{3}$ m (4) $1200\sqrt{3}$ m

Ans. **(4)**

Sol.



$v = 432 \times \frac{1000}{60\times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In ΔPAC

$\tan 60° = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$

In ΔPBD

$\tan 30° = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$

$PD = PC + CD$

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

$h = 1200\sqrt{3}$ meter

6. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2)$ is:

(1) $\frac{n(n-1)(2n+1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$ (3) $\frac{n(2n+1)(3n+1)}{6}$ (4) $\frac{n(n+1)^2(n+2)}{12}$

Ans. **(2)**

Sol. $S = {}^2C_2 + {}^3C_2 + \ldots\ldots + {}^nC_2 = {}^{n+1}C_3$

$\therefore\ {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \frac{(n+1)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$

$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)}{6}(2n+1)$

7. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as,

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let $A = \{x \in \mathbf{R} : f \text{ is increasing}\}$. Then A is equal to :

(1) $(-\infty, -5) \cup (4, \infty)$ (2) $(-5, \infty)$

(3) $(-\infty, -5) \cup (-4, \infty)$ (4) $(-5, -4) \cup (4, \infty)$

Ans. **(4)**

Sol. $$f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x^2 - x - 20) & ; \quad -5 < x < 4 \\ 6(x^2 - x - 6) & ; \quad x > 4 \end{cases}$$

$$f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x-5)(x+4) & ; \quad -5 < x < 4 \\ 6(x-3)(x+2) & ; \quad x > 4 \end{cases}$$

Hence, f(x) is monotonically increasing is $(-5, -4) \cup (4, \infty)$

8. Let f be a twice differentiable function defined on R such that f(0) = 1, f '(0) = 2 and f '(x) ≠ 0 for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of f(1) lies in the interval:

(1) (9, 12) (2) (6, 9) (3) (0, 3) (4) (3,6)

Ans. **(2)**

Sol. Given $f(x)\, f''(x) - (f'(x))^2 = 0$

Let $h(x) = \frac{f(x)}{f'(x)}$

$\Rightarrow h'(x) = 0 \qquad \Rightarrow h(x) = k$

$\Rightarrow \frac{f(x)}{f'(x)} = k \qquad \Rightarrow f'(x) = k\, f'(x)$

$\Rightarrow f(x) = k\, f'(0) \qquad \Rightarrow 1 = k(2) \Rightarrow k = \frac{1}{2}$

New $f(x) = \frac{1}{2} f'(x) \Rightarrow \int 2dx = \int \frac{f'(x)}{f(x)} dx$

$\Rightarrow 2x = \ln|f(x)| + C$

As $f(0) = 1 \Rightarrow C = 0$

$\Rightarrow 2x = \ln|f(x)| \Rightarrow f(x) = \pm e^{2x}$

As $f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$

9. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + y^2 = 7$ (2) $y^2 = \frac{1}{6\sqrt{3}}x$ (3) $2x^2 - 18y^2 = 9$ (4) $x^2 + 9y^2 = 9$

Ans. **(4)**

Sol. tangent to $x^2 + 9y^2 = a$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ is $x\left(\frac{3\sqrt{3}}{2}\right) + 9y\left(\frac{1}{2}\right) = 9$

$\Rightarrow$ option (4) is true

10. The value of the integral, $\int_1^3 [x^2 - 2x - 2]\, dx$, where [x] denotes the greatest integer less than or equal to x, is :

(1) $-\sqrt{2} - \sqrt{3} + 1$ (2) $-\sqrt{2} - \sqrt{3} - 1$ (3) –5 (4) –4

Ans. **(2)**

Sol. $I = \int_1^3 -3dx + \int_1^3 [(x-1)^2]\, dx$ $\qquad x - 1 = t; dx = dt$

$I = (-6) + \int_0^2 [t^2] dt$

$I = -6 + \int_0^1 0\, dt + \int_1^{\sqrt{2}} 1\, dt + \int_{\sqrt{2}}^{\sqrt{3}} 2\, dt + \int_{\sqrt{3}}^{2} 3\, dt$

$I = -6 + \left(\sqrt{2}-1\right) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

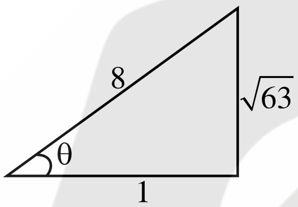
$I = -1 - \sqrt{2} - \sqrt{3}$

11. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

(1) $\frac{1}{\sqrt{7}}$ (2) $2\sqrt{2}-1$ (3) $\sqrt{7}-1$ (4) $\frac{1}{2\sqrt{2}}$

Ans. **(1)**

Sol. Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{63}}{8}$



$\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \tan\left(\frac{\theta}{4}\right) = \frac{1-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \frac{1-\sqrt{\frac{1+\cos\theta}{2}}}{\sqrt{\frac{1-\cos\theta}{2}}} = \frac{1-\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{1}{\sqrt{7}}$

12. The negative of the statement $\sim p \wedge (p \vee q)$ is

(1) $\sim p \vee q$ (2) $p \vee \sim q$ (3) $\sim p \wedge q$ (4) $p \wedge \sim q$

Ans. **(2)**

Sol. $\sim\left(\sim p \wedge \left(p \vee q\right)\right)$

$= \sim\left(\left(\sim p \wedge p\right) \vee \left(\sim p \wedge q\right)\right)$

$= \sim\left(\sim p \wedge q\right) = p \vee \sim q$

13. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are :

(1) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$ (2) $a = 1, b = 0, c = 1$

(3) $a = 1, b = 1, c = 0$ (4) $a = -1, b = 1, c = 1$

Ans. **(3)**

Sol. $2 = a + b + c$ .... (i)

$\frac{dy}{dx} = 2ax + b \Rightarrow \left.\frac{dy}{dx}\right|_{(0,0)} = 1$

$\Rightarrow b = 1 \Rightarrow a + c = 1$

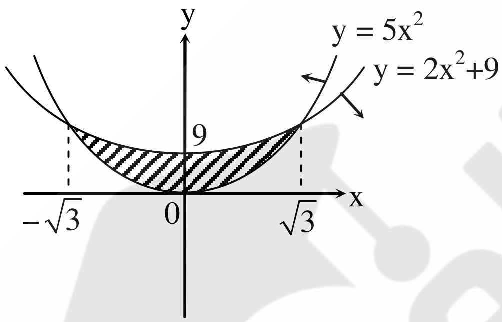
14. The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is :

(1) $11\sqrt{3}$ square units (2) $12\sqrt{3}$ square units

(3) $9\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

Ans. **(2)**

Sol.



Required area

$= 2\int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2)dx$

$= 2\int_0^{\sqrt{3}} (9 - 3x^2)dx$

$= 2\left|9x - x^3\right|_0^{\sqrt{3}} = 12\sqrt{3}$

15. If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

(1) 5 (2) 10 (3) $\frac{62}{5}$ (4) $\frac{31}{5}$

Ans. **(2)**

Sol. $\frac{dy}{dx}+\frac{y}{x}=6x^3$

I.F. = $e^{\int\frac{dx}{x}}=x$

$\therefore$ yx = $\int bx^4 dx=\frac{bx^5}{5}+C$

Passes through (1,2) , we get

$2=\frac{b}{5}+C$ ..(i)

Also , $\int_1^2\left(\frac{bx^4}{5}+\frac{C}{x}\right)dx=\frac{62}{5}$

$\Rightarrow \frac{b}{25}\times 32+C\ell n2-\frac{b}{25}=\frac{62}{5}$

$\Rightarrow$ C = 0 & b = 10

16. Let f(x) be a differentiable function defined on [0, 2] such that f '(x) = f '(2 – x) for all x $\in$ (0, 2), f(0) = 1 and f(2) = e^2. Then the value of $\int_0^2 f(x)\,dx$ is :

(1) $1-e^2$ (2) $1+e^2$ (3) $2(1-e^2)$ (4) $2(1+e^2)$

Ans. **(2)**

Sol. $f'(x)=f'(2-x)$

On integrating both side $f(x)=-f(2-x)+c$

put x = 0

$f(0)+f(2)=c \Rightarrow c=1+e^2$

$\Rightarrow f(x)+f(2-x)=1+e^2$……(i)

$I=\int_0^2 f(x)\,dx=\int_0^1\{f(x)+f(2-x)\}\,dx=(1+e^2)$

17. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2-B^2A^2)X=O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × l null matrix, has :

(1) no solution

(2) exactly two solutions

(3) infinitely many solutions

(4) a unique solution

Ans. **(3)**

Sol. $A^T = A, B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix

$\Rightarrow |P| = 0$

Hence PX = 0 have infinite solution

18. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is :

(1) $\frac{71}{256}$ (2) $\frac{69}{256}$ (3) $-\frac{69}{256}$ (4) $-\frac{71}{256}$

Ans. **(4)**

Sol. $2b = a + c$

$\frac{2a+2}{3} = \frac{10}{3}$ and $\frac{2b+c}{3} = \frac{7}{3}$

$\Rightarrow a = 4$ $\left.\begin{matrix} 2b+c=7 \\ 2b-c=4 \end{matrix}\right\}$ solving,

$b = \frac{11}{4}$ $c = \frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2 + \frac{11}{4}x + 1 = 0$

$\therefore$ The value of $(\alpha+\beta)^2 - 3\alpha\beta = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$

19. For the system of linear equations :

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$,

consider the following statements :

(A) The system has unique solution if $k \neq 2$, $k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct ?

(1) (C) and (D) only (2) (B) and (E) only

(3) (A) and (E) only (4) (A) and (D) only

Ans. **(4)**

Sol. $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$

$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = -(k+10)(k+2)$

for unique solution $D \neq 0 \Rightarrow k \neq \pm 2$

for no solution, $D = 0$, $D_1 \neq 0$

$\Rightarrow k = 2$

at $k = -2$, $D_1 = D_2 = D_3 = D = 0$ So system has infinite solution.

20. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

(1) $\frac{65}{2^7}$ (2) $\frac{65}{2^8}$ (3) $\frac{135}{2^9}$ (4) $\frac{35}{2^7}$

Ans. **(3)**

Sol. Required probability

$= \frac{{}^5C_2 \times 3^3}{4^5}$

$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$

SECTION-B

1. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$ The maximum value of k for which the sum $\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to ________.

Ans. **(12)**

Sol. $\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i}$

$= {}^{10}C_0 \cdot {}^{15}C_k + {}^{10}C_1 \cdot {}^{15}C_{k-1} + {}^{10}C_2 \cdot {}^{15}C_{k-2} + \ldots\ldots + {}^{10}C_k \cdot {}^{15}C_0$

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + {}^{10}C_3 x^3 + \ldots\ldots + {}^{10}C_k x^k + \ldots\ldots$

$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1 x + {}^{15}C_2 x^2 + \ldots\ldots + {}^{15}C_k x^k + \ldots\ldots$

Now

$(1 + x)^{10}, (1 + x)^{15} = (\qquad) (\qquad)$

compare coeff. of x^k both side

$^{25}C_k = {}^{10}C_0\ {}^{15}C_k + ... + {}^{10}C_k.{}^{15}C_0$

Similar $\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i} = {}^{25}C_{k+1}$

Now $^{25}C_k + {}^{25}C_{k+1}$

$= {}^{26}C_{k+1}$

to find the maximum sum value of k is 12

* but as per question maximum value of k is infinite

So, it should be BONUS

2. Let λ be an interger. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _______.

Ans. **(1)**

Sol. $L_1 : \frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$ and

$L_2 : \frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$

$$D\phi = \left|\frac{(\vec{a}_1 - \vec{a}_2).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix}$$

$= \hat{i}(1) - \hat{j}\left(\frac{3}{2}\right) + \hat{k}\left(\frac{1}{2}\right)$

$= \hat{i} - \frac{3\hat{j}}{2} + \frac{\hat{k}}{2}$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{1 + \frac{9}{4} + \frac{1}{4}} = \sqrt{\frac{7}{2}}$

$|\vec{a}_1 - \vec{a}_2| = \lambda\hat{i} + \left(\frac{1}{2} + 2\lambda\right)\hat{j} - \lambda\hat{k}$

given

$$D = \left| \frac{\lambda - \left(\frac{3}{4} + 3\lambda\right) - \frac{\lambda}{2}}{\sqrt{\frac{7}{2}}} \right| = \sqrt{\frac{7}{2}} \times \frac{1}{2}$$

$$\left|\frac{5\lambda}{2} + \frac{3}{4}\right| = \frac{7}{4} \Rightarrow \lambda = \frac{2}{5}, \lambda = -1$$

$\lambda \in$ integer

$|\lambda| = 1$

3. If $a + \alpha = 1$, $b + \beta = 2$ and

$af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \neq 0$, then the value of expression $\dfrac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is ________.

Ans. **(2)**

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ .... (i)

$x \to \frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ .... (ii)

(i) + (ii)

$$(a + \alpha)\left[f(x) + f\left(\frac{1}{x}\right)\right] = \left(x + \frac{1}{x}\right)(b + \beta)$$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

4. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (– 5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ________.

Ans. **(56)**

Sol. Internal point which divide (5,0) & (–5,0) in the ratio 3 : 1 is $\left(\frac{-5}{2}, 0\right)$ External point which divide (5,0) & (–5,0) in the ratio 3 : 1 is (–10,0)

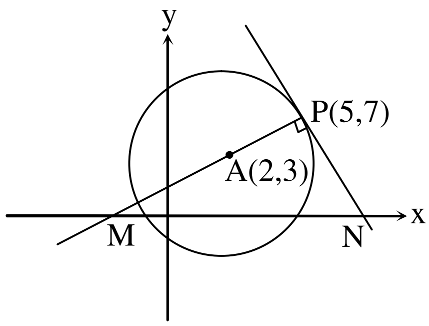
$$2r = \left(\frac{-5}{2} + 10\right) = \frac{15}{2} = 7.5$$

$$(2r)^2 = 56.25 \approx 56$$

5. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to_____.

***Ans.** **(1225)**

Sol.



equation of normal at P

$(y-7) = \left(\frac{7-3}{5-2}\right)(x-5)$

$3y - 21 = 4x - 20$

$\Rightarrow 4x - 3y + 1 = 0$ .... (i)

$\Rightarrow M\left(-\frac{1}{4}, 0\right)$

equation of tangent at P

$(y-7) = -\frac{3}{4}(x-5)$

$4y - 28 = -3x + 15$

$\Rightarrow 3x + 4y = 43$ .... (ii)

$\Rightarrow N\left(\frac{43}{3}, 0\right)$

hence ar(ΔPMN) = $\frac{1}{2} \times MN \times 7$

$l = \frac{1}{2} \times \frac{175}{12} \times 7$

$\Rightarrow 24\lambda = 1225$

* **(In the question positive x-axis is given which do not from any triangle for required area)**

6. If the variance of 10 natural numbers 1, 1, 1,...., 1, k is less than 10, then the maximum possible value of k is ______.

Ans. **(11)**

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\sigma^2 = \frac{(9+k^2)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$(90 + k^2)10 - (81 + k^2 + 8k) < 1000$

$90 + 10k^2 - k^2 - 18k - 81 < 1000$

$9k^2 - 18k + 9 < 1000$

$(k-1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

7. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _______.

Ans. **(3)**

Sol. a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$ ...... (i)

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$\frac{1}{a}\left(\frac{r^3 + r^2 + r + 1}{r^3}\right) = \frac{65}{18}$ .... (ii)

$\frac{(i)}{(ii)}, a^2r^3 = \frac{18}{12} = \frac{3}{2}$

$a^3r^3 = 1 \Rightarrow a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$

$\frac{4}{9}r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$

$\alpha = ar^2 = \frac{2}{3}.\left(\frac{3}{2}\right)^2 = \frac{3}{2}$

$2\alpha = 3$

8. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Ans. **(31650)**

Sol.

A	B	C
1	8	1
2	7	1
⋮	⋮	⋮
6	1	3

Ways to distribute in groups = ${}^{10}C_1({}^9C_1 + \ldots\ldots. {}^9C_8) + {}^{10}C_2({}^8C_1 + \ldots..+ {}^8C_7) + {}^{10}C_3({}^7C_1 + \ldots.. + {}^7C_6)$

= 10(510) + 45(254) + 120(126)

= 31650

9. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}$ = k, and n = [| k |] be the greatest integral part of | k |. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to ________.

Ans. **(310)**

Sol. $\frac{\left(-1+i\sqrt{3}\right)^{21}}{\left(-2i\right)^{12}}+\frac{\left(1+i\sqrt{3}\right)^{21}}{\left(2i\right)^{12}}=K$

$\Rightarrow \frac{1}{2^{12}}\left[\left(1+i\sqrt{3}\right)^{21}-\left(1-i\sqrt{3}\right)^{21}\right]=K$

$\Rightarrow \frac{1}{2^{12}}\left[2^{21}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{21}-2^{21}\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^{21}\right]=k\Rightarrow 2^9\left(2\sin\frac{21\pi}{3}\right)=k\Rightarrow k=0$

now n = [|k|] = 0

then $\sum_{J=0}^{5}\left(j+5\right)^2-\sum_{J=0}^{5}\left(j+5\right)=\sum_{J=0}^{5}\left(j+5\right)^2-\left(j+5\right)$

$\sum_{J=0}^{5}\left(J^2+9J+20\right)=20+\left(\sum_{J=1}^{5}J^2+9\sum_{J=1}^{5}J+20\sum_{J=1}^{5}1\right)$

= 310

10. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is ________.

Ans. **(2)**

Sol. $(x+1)^2+|x-5|=\frac{27}{4}$

(i) $x \geq 5$

$x^2+3x-4=\frac{27}{4}$

$4x^2+12x-43=0$

$D \geq 0$

$x=\frac{-12\pm\sqrt{144+16\times 43}}{8}$

for $x=\frac{-3\pm\sqrt{52}}{2}$

for $x \geq 5$, both not possible

(ii) $x < 5$

$x^2+x+6=\frac{27}{4}$

$4x^2+4x-3=0$

$x=\frac{-4\pm\sqrt{16+48}}{8}$

$x=\frac{1}{2}$, $x=\frac{-3}{2}$ both possible for $x < 5$