JEE MAIN 2021

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PAPER-1 (B.E. / B.TECH)

QUESTIONS & SOLUTIONS

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17 March, 2021 SHIFT-1

09:00 am to 12 Noon

Duration : 3 Hours Max. Marks : 300

SUBJECT - MATHEMATICS

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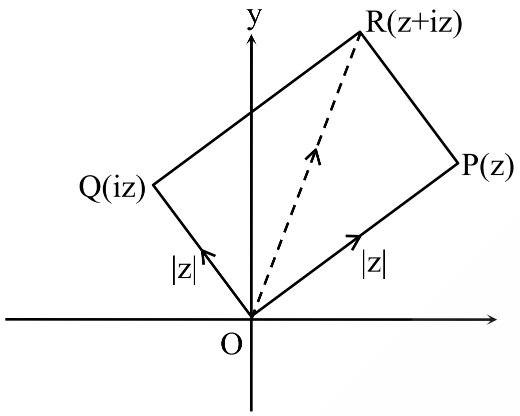
MATHEMATICS

1. Let z, iz, z + iz, are vertices of the triangle then area of this triangle is

(1) $\frac{1}{2}|z|^2$ (2) $\frac{1}{2}|z + iz|^2$ (3) 0 (4) 1

Ans. (1)

Sol.



Area of $\Delta = \frac{1}{2}$ (area of square) $= \frac{1}{2}|z|^2$

2. Let $4x + 3y \le 75$, $3x + 4y \le 100$, $x \ge 0$, $y \ge 0$ and $z = 6xy + y^2$. Find maximum value z is

(1) $\frac{(255)^2}{56}$ (2) $\frac{(255)^2}{28}$ (3) $\frac{(225)^2}{56}$ (4) $\frac{(225)^2}{28}$

Ans. (3)

Sol. $4x + 3y \le 75$, $3x + 4y \le 100$

$z = 6xy + y^2$, $x \ge 0$, $y \ge 0$

$\Rightarrow x = \frac{z - y^2}{6y}$

$\Rightarrow 4\left(\frac{z-y^2}{6y}\right) + 3y \le 75$ and $3\left(\frac{z-y^2}{6y}\right) + 4y \le 100$

$\Rightarrow z \le 200y - 7y^2$ and $z \le \frac{225y - 7y^2}{2}$

Range of $200y - 7y^2$ and $\frac{225y - 7y^2}{2}$

are $\left(-\infty, \frac{10^4}{7}\right]$ and $\left(-\infty, \frac{225^2}{8 \times 7}\right]$

respectively

$\Rightarrow$ Maximum $z = \min\left(\frac{10^4}{7}, \frac{225^2}{56}\right)$

$\Rightarrow$ Maximum $z = \frac{225^2}{56}$

3. There are two dice each numbers as 1, 2, 3, 5, 7, 11. Find the probability that the sum of the no's on them is less than or equal to 8.

(1) $\frac{1}{2}$ (2) $\frac{17}{36}$ (3) $\frac{19}{36}$ (4) none of these

Ans. (2)

Sol. n(S) = 36

possible ordered pair ; (1, 1), (1, 2), (1, 3), (1, 5), (1, 7), (2, 1), (2, 2), (2, 3), (2, 5), (3, 1), (3, 2), (3, 3), (3, 5), (5, 1), (5, 2), (5, 3), (7, 1)

Number of ordered pair = 17

Probability = $\frac{17}{36}$

4. Let $(p \to q) \leftrightarrow (\sim q * p)$ is a tautology, then $p * \sim q$ is equivalent to -

(1) $p \to q$ (2) $p \vee q$ (3) $p \leftrightarrow q$ (4) $p \wedge q$

Ans. (1)

Sol.

p	q	~q	$p \to q$	~q * p	$\sim q \wedge p$	$\sim(\sim q \wedge p)$
T	T	F	T	T	F	T
F	T	F	T	T	F	T
T	F	T	F	F	T	F
F	F	T	T	T	F	T

for $(p \to q) \leftrightarrow (\sim q * p)$ to be tautology, truth values of ~q * p will be as shown in table so.

$(\sim q * p) \equiv \sim(\sim q \wedge p) \equiv q \vee \sim p \equiv \sim p \vee q \equiv p \to q$

Hence $p * \sim q \equiv \sim(p \wedge \sim q) \equiv \sim p \vee q \equiv p \to q$

5. The value of $\lim_{x \to 0^+} \frac{\cos^{-1}(x-[x]^2)\sin^{-1}(x-[x]^2)}{x-x^3}$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $-\frac{\pi}{2}$ (4) $-\frac{\pi}{4}$

Ans. (1)

Sol. $\lim_{x \to 0^+} \frac{\cos^{-1}(x-[x]^2)\sin^{-1}(x-[x]^2)}{x(1-x^2)}$

$\Rightarrow \lim_{x \to 0^+} \frac{\cos^{-1}x \ \sin^{-1}x}{x} = \frac{\pi}{2}$

6. If $\frac{dy}{dx} = (x-1)y + x - 1$, y (0) = 0, find y(1) =

(1) $e^{\frac{1}{2}} - 1$ (2) $e^{-\frac{1}{2}} - 1$ (3) $1 - e^{-\frac{1}{2}}$ (4) $1 + e^{-\frac{1}{2}}$

Ans. (2)

Sol. $\frac{dy}{dx} = (x-1)y + (x-1)$

$\frac{dy}{dx} = (x-1)(y+1)$

$\frac{dy}{y+1} = (x-1)\,dx$

$\ell n(y+1) = \frac{x^2}{2} - x + c$

$x = 0,\ y = 0$

$\Rightarrow c = 0$

$\therefore \ \ell n(y+1) = \frac{x^2}{2} - x$

putting $x = 1$, $\ell n(y+1) = \frac{1}{2} - 1 = -\frac{1}{2}$

$y + 1 = e^{-\frac{1}{2}}$

$y = e^{-\frac{1}{2}} - 1$

$\therefore \ y(1) = e^{-\frac{1}{2}} - 1$

7. The value of $\int_0^{\sqrt{\pi/2}} [[x^2] + \cos x]\,dx$; (where [.] denotes greatest integer function)

(1) $1-\sqrt{\frac{\pi}{2}}$ (2) $\sqrt{\frac{\pi}{2}}$ (3) $\sqrt{\frac{\pi}{2}}+1$ (4) $\sqrt{\frac{\pi}{2}}-1$

Ans. (4)

Sol. $I = \int_0^1 [\cos x]dx + \int_1^{\sqrt{\pi/2}} [1 + \cos x]dx$

$= \int_0^1 0 + \int_1^{\sqrt{\pi/2}} 1\,dx + \int_1^{\sqrt{\pi/2}} [\cos x]dx$

$= 0 + \sqrt{\frac{\pi}{2}} - 1 + \int 0\,dx$

$= \sqrt{\frac{\pi}{2}} - 1$

8. If 4^{th} term in the expansion of $\left(x+x^{\log_2 x}\right)^7$ is 4480 then x is equal to

(1) 2 (2) 3 (3) 4 (4) 5

Ans. (1)

Sol. $^7C_3\ x^4\left(x^{\log_2 x}\right)^3 = 4480$

$35\ x^4\left(x^{\log_2 x}\right)^3 = 4480$

$x^4\left(x^{\log_2 x}\right)^3 = 128$

take log w.r.t base 2 we get $4\log_2 x + 3\log_2\left(x^{\log_2 x}\right) = \log_2 128$

Let $\log_2 x = y$

$4y + 3y^2 = 7$

$\Rightarrow y = 1,\ \frac{-7}{3}$

$\Rightarrow \log_2 x = 1,\ \frac{-7}{3}$

$x = 2,\ x = 2^{-7/3}$

9. Let $g(\alpha) = \int_{\pi/6}^{\pi/3}\frac{\sin^\alpha x}{\sin^\alpha x+\cos^\alpha x}dx$ than which of the following option is **CORRECT** ?

(1) $g(\alpha)$ is strictly increasing function

(2) $g(\alpha)$ is strictly decreasing function

(3) $g(\alpha)$ is even function

(4) $g(\alpha)$ has point of inflection at $\alpha = -\frac{1}{2}$

Ans. (3)

Sol. $g(\alpha) = \int_{\pi/6}^{\pi/3}\frac{\sin^\alpha x}{\sin^\alpha x+\cos^\alpha x}dx$ .... (i)

$g(\alpha) = \int_{\pi/6}^{\pi/3}\frac{\cos^\alpha x}{\sin^\alpha x+\cos^\alpha x}dx$ .... (ii)

adding equation (i) and (ii)

$2g(\alpha) = \int_{\pi/6}^{\pi/3}1\,dx = \frac{\pi}{6}$

$\Rightarrow g(\alpha) = \frac{\pi}{12}$

10. If $y = 4 + \cfrac{1}{5+\cfrac{1}{4+\frac{1}{5}+\cfrac{1}{4+\frac{1}{5}+......}}}$ then find the value of 'y'

(1) $\frac{10+2\sqrt{30}}{5}$ (2) $\frac{10-2\sqrt{30}}{5}$ (3) $\frac{5+\sqrt{30}}{10}$ (4) $\frac{6-\sqrt{30}}{5}$

Ans. (1)

Sol. $y = 4 + \cfrac{1}{5+\cfrac{1}{y}}$

$y = 4 + \frac{y}{5y+1}$

$5y^2 - 20y - 4 = 0$

$y = \frac{20 \pm \sqrt{400+80}}{10}$

$y = \frac{20 \pm 4\sqrt{30}}{10}$, $y > 0$

$y = \frac{10+2\sqrt{30}}{5}$

11. If $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4} = \frac{1}{k}$ then find the value of k.

(1) 4 (2) 6 (3) 2 (4) 3

Ans. (2)

Sol. $\frac{1}{k} = \lim_{x\to 0}\frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$

$= \lim_{x\to 0}\frac{(\sin x + x)(x-\sin x)}{2x^4} = \left(\lim_{x\to 0}\frac{\sin x + x}{2x}\right)\left(\lim_{x\to 0}\frac{x-\sin x}{x^3}\right)$

$= 1\times \lim_{x\to 0}\frac{1-\cos x}{3x^2} = \frac{1}{6}$

$k = 6$

12. y-axis lies on a plane having point (1, 2, 3), then equation of plane is

(1) $x + 3z = 10$ (2) $x + 3z = 0$ (3) $3x - z = 0$ (4) $3x + y = 6$

Ans. (3)

Sol. Let the equation of the plane is a(x – 1) + b(y – 2) + c (z – 3) = 0

y-axis lies on it D.R.'s of y-axis are 0, 1, 0

$\therefore$ 0.a + 1.b + 0.c = 0 $\Rightarrow$ b = 0

$\therefore$ Equation of plane is a(x – 1) + c (z – 3) = 0

x = 0, z = 0 also satisfy it –a – 3c = 0 $\Rightarrow$ a = –3c

–3c (x – 1) + c (z – 3) = 0

–3x + 3 + z – 3 = 0

3x – z = 0

13. If $A = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}$ where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\left|A^2 - \frac{1}{2}I\right| = 0$, then the value of α is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (4)

Sol. $A^2 = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}\begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix} = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix}$

$A^2 - \frac{1}{2}I = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{bmatrix}$

$\Rightarrow \left|A^2 - \frac{1}{2}I\right| = 0$

$\begin{vmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{vmatrix} = 0$

$\Rightarrow \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0 \Rightarrow \sin^2\alpha = \frac{1}{2}$ $\Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

$\alpha = \frac{\pi}{4}$

14. If $\tan^{-1}(1+x) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\frac{8}{31}$, then sum of possible value of 'x' is equal to

(1) $\frac{-31}{4}$ (2) $\frac{-33}{4}$ (3) $\frac{-32}{4}$ (4) $\frac{-30}{4}$

Ans. (3)

Sol. Taking tan both sides

$$\frac{(1+x)+(x-1)}{1-(1+x)(x-1)}=\frac{8}{31}$$

$$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow x = -8, \frac{1}{4}$

but at $x = \frac{1}{4}$

LHS $> \frac{\pi}{2}$ and RHS $< \frac{\pi}{2}$

So, only solution is $x = -8$

15. If the equation

$kx + y + z = 1$

$x + ky + z = 1$

$x + y + kz = 1$

have no solution, then

(1) $k = 1$ (2) $k = -2$ (3) $k = -1$ (4) $k = 2$

Ans. (2)

Sol. $D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$

$k(k^2 - 1) - (k - 1) + (1 - k) = 0$

$(k - 1)(k^2 + k - 1 - 1) = 0$

$(k - 1)(k^2 + 1 - 2) = 0$

$(k - 1)(k - 1)(k + 2) = 0$

$k = 1, k = 2$

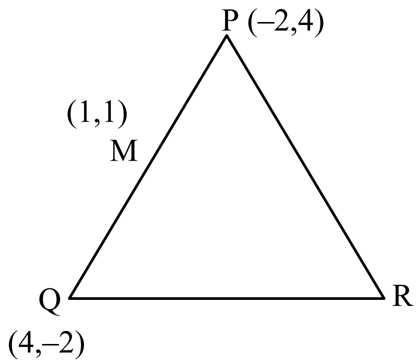
for $k = 1$ equation identical so $k = -2$ for no solution.

16. If PQR is triangle having vertices P(–2, 4) and Q(4, –2) and perpendicular bisector of PR is $2x - y + 2 = 0$ then circumcenter of ΔPQR is

(1) (1,4) (2) (–2,–2) (3) (2,6) (4) (0,2)

Ans. (2)

Sol.



Perpendicular bisector of PR : $2x - y + 2 = 0$ .....(1)

Mid point of PQ $\rightarrow$ M(1,1)

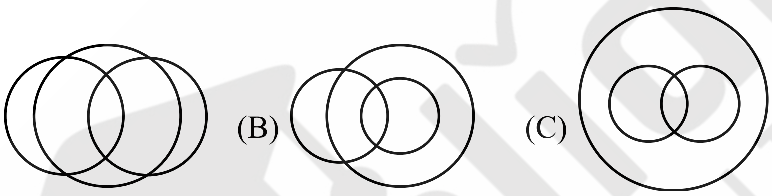
equation of perpendicular bisector of PQ : $x - y = 0$ ......(2)

$\therefore$ POI of equation (1) & (2) is circumcentre

So, circumcentre (–2,–2)

17. 3 games are played in a school. If some students played exactly 2 games, and no student play all the 3 games, then which venn diagram can represents the above situation

(A) (B) (C)



(1) Only A is correct (2) A & B are correct

(3) Only C is correct (4) None of these

Ans. (4)

Sol. In are the (A), (B), (C) there are some students which play all the three games hence no venn diagram is correct

18. Let there are two teams, one team has 7 boys and 6 girls & other team has 4 boys n girls. Let total match between two teams of same genders one 52 (each match is happened between two players) then n is equal to

(1) 8 (2) 6 (3) 5 (4) 4

Ans. (4)

Sol. $7 \times 4 + 6 \times n = 52$

$6n = 24$

$\Rightarrow n = 4$

19. Let $S_1 \equiv x^2 + y^2 - 10x - 10y - + 41 = 0$ and $S_2 \equiv x^2 + y^2 - 16x - 10y + 80 = 0$ are two circles, then which of the following is INCORRECT ?
(1) Both circles intersect each other at two distinct points
(2) Centres of both circles lie inside the region of each other
(3) Distance between centres is average of radii of circles
(4) Both circles pass through centers of each other

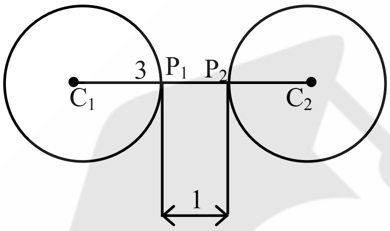
Ans. (2)

Sol. C_1 (5, 5), C_2 (8, 5)
position of $C_1(5, 5)$ in $S_2 = 0$
$= 25 + 25 - 80 - 50 + 80$
$= 0$
position of $C_2(8, 5)$ in $S_1 = 0$
$\equiv 64 + 25 - 80 - 50 + 41$
$= 0$
$\Rightarrow S_1$ and S_2 intersect each other and pass through center of each other

20. Let S_1 & S_2 be two circles $x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 - 24x - 10y + 160 = 0$ respectively P_1 is a point on S_1 and P_2 is a point of S_2. Minimum value of distance P_1P_2 is ______

Ans. 1

Sol. $S_1 : (x - 5)^2 + (y - 5)^2 = 9$ centre (5, 5), $r_1 = 3$
$S_2 : (x - 12)^2 + (y - 5)^2 = 9$ centre (12, 5), $r_2 = 3$



So $(P_1P_2)_{min} = 1$

21. If $\cot^{-1}\alpha = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + ...$ up to 100 terms then 'α' is equal to

Ans. 1.01

Sol. $\text{RHS} = \sum_{n=1}^{100} \cot^{-1} 2n^2 = \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$= \tan^{-1}201 - \tan^{-1}1$

$$= \tan^{-1}\left(\frac{200}{202}\right)$$

$$\Rightarrow \cot^{-1}\alpha = \cot^{-1}\left(\frac{101}{100}\right)$$

$\Rightarrow \alpha = 1.01$

22. Let B_i (i = 1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\ p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0,1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to______.

Ans. 6

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow\ x(1-y)(1-z) = \alpha \Rightarrow\ y(1-x)(1-z) = \beta \Rightarrow\ z(1-x)(1-y) = \gamma \Rightarrow (1-x)(1-y)\,(1-z) = P$

Putting in the given relation we get x = 2y and y = 3z $\Rightarrow$ x = 6z $\Rightarrow$ $\frac{x}{z} = 6$

23. Let $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, the value of $\det(A^4) - \det(A^{10} - (\text{adj}(2A))^{10})$

Ans. 16

Sol. $|A| = -2 \Rightarrow |A|^4 = 16$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10}-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$\text{adj}(2A) = -2\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$$

$$(\text{adj}(2A))^{10} = 2^{10}\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$$

$$= 2^{10}\begin{bmatrix} 1 & -\left(2^{10}-1\right) \\ 0 & 2^{10} \end{bmatrix}$$

$$= 2^{10}\begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$$

$$A^{10} - (\text{adj}(2A))^{10} = \begin{bmatrix} 0 & 2^{11}\times 1023 \\ 0 & 1-(1024)^2 \end{bmatrix}$$

$|A^{10}\ \text{adj}(2A)^{10}| = 0$

24. Find the remainder when $(2021)^{3762}$ is divided by 17.

Ans. 4

Sol. $(2021)^{3762} = (2023 - 2)^{3762}$ = multiple of 17 + 2^{3762}

$= 17\lambda + 2^2\,(2^4)^{940}$

$= 17\lambda + 4\,(17 - 1)^{940}$

$= 17\lambda + 4\,(17\mu + 1)$

$17k + 4;\ (k \in I)$

Remainder = 4

25. If $\vec{r}\times\vec{a} = \vec{r}\times\vec{b}$, $\vec{a} = 2\hat{i} - 3\hat{j} + 4k$, $\vec{b} = 7\hat{i} + \hat{j} - 6k$, $\vec{c} = \hat{i} + 2\hat{j} + k$, and $\vec{r}.\vec{c} = -3$ find $\vec{r}.\vec{a}$

Ans. 42

Sol. $\vec{r}\times\vec{a} - \vec{r}\times\vec{b} = \vec{0}$

$\vec{r}\times\left(\vec{a} - \vec{b}\right) = \vec{0}$

$\Rightarrow \vec{r} = \lambda\left(\vec{a} - \vec{b}\right)$

$\Rightarrow \vec{r} = \lambda\left(-5\hat{i} - 4\hat{j} + 10\hat{k}\right)$

$\vec{r}.\vec{c} = -3 \Rightarrow \lambda\left(-5 - 8 + 10\right) = -3$

$\Rightarrow \lambda = 1$

$\therefore\ \vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$\vec{r}.\vec{a} = \left(-5\hat{i} - 4\hat{j} + 10\hat{k}\right).\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$

$= -10 + 12 + 40 = 42$

26. Let $2x - 7y + 4z - 11 = 0$ and $-3x - 5y + 4z - 3 = 0$ are two planes. If plane $ax + by + cz - 7 = 0$ passes through the line of intersection of given planes and point $(-2, 1, 3)$, then find the value of $2a + b + c + 7$.

Ans. 4

Sol. Equation of plane is $(2x - 7y + 4z - 11) + \lambda(-3x - 5y + 4z - 3) = 0$

it passes through the point $(-2, 1, 3)$

$\therefore\ (-4 - 7 + 1) + \lambda(6 - 5 + 9) = 0 \qquad \Rightarrow \lambda = 1$

$\therefore$ Equation of plane is $-x - 12y + 8z - 14 = 0$

$\Rightarrow -\frac{1}{2}x - 6y + 4z - 7 = 0$

$\therefore\ a = -\frac{1}{2},\ b = -6,\ c = 4$

$\therefore\ 2a + b + c + 7 = 4$