

QUESTIONS & SOLUTIONS

Reproduced from Memory Retention

📅 16 March, 2021

🕒 09:00 am to 12 Noon

SHIFT-1



Duration : 3 Hours

Max. Marks : 300

SUBJECT - MATHEMATICS

JEE (MAIN) FEB 2021 RESULT

Legacy of producing
Best Results Proved again

RELIABLE
TOPPER



100%tile
in **MATHS**

PRANAV JAIN
Roll No. : 20771421
99.993%tile
Overall

100%tile
in **MATHS & PHYSICS**

KHUSHAGRA GUPTA
Roll No. : 20975433

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21 Students
Secured
100%tile
in Maths / Physics

138
students secured
above **99%**tile (Overall)

All are from **KOTA CLASSROOM** only



TARGET
JEE (MAIN+ADV.)
2021

SHAKTI
COMPACT COURSE

for XII passed students

Course
Duration
250+
Hrs

Starting from



22nd MAR
2021

Course will be available in both
Offline & Online mode

MATHEMATICS

1. Three distinct normal are drawn from the point $(a, 0)$ to the parabola $y^2 = 2x$. The range of 'a' is
 (1) $(-\infty, 0)$ (2) $(1, \infty)$ (3) $(-\infty, -1)$ (4) $(0, 1)$

Ans. (2)

Sol. Let the equation of the normal is

$$y = mx - 2am - am^3$$

here $4a = 2 \Rightarrow a = \frac{1}{2}$

$$y = mx - m - \frac{1}{2}m^3$$

It passing through $A(a, 0)$ then

$$0 = am - m - \frac{1}{2}m^3$$

$$m = 0, a - 1 - \frac{1}{2}m^2 = 0$$

$$m^2 = 2(a - 1) > 0$$

$$\therefore a > 1$$

2. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$. Then the equation has

- (1) one solution (2) two solution (3) infinite solution (4) no solution

Ans. (4)

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$$A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^4 = 4 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = 4 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8 = 64 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 64 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$128 \begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \Rightarrow 128(x-y) = 8$$

$$\Rightarrow x-y = \frac{1}{16} \quad \dots(1)$$

$$\text{and } 128(-x+y) = 64 \Rightarrow x-y = \frac{-1}{2} \quad \dots(2)$$

\Rightarrow no solution

3. If $x^2 + y^2 = 25$ is a circle whose chord is a tangent to a hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then locus of mid-point of chord is

(1) $9x^2 + 16y^2 = (x^2 + y^2)^2$

(2) $9x^2 - 25y^2 = (x^2 - y^2)^2$

(3) $9x^2 - 16y^2 = (x^2 + y^2)^2$

(4) $9x^2 + 25y^2 = (x^2 + y^2)^2$

Ans. (3)

Sol. tangent of hyperbola

$$y = mx \pm \sqrt{9m^2 - 16} \dots\dots(i)$$

which is a chord of circle with mid-point (h, k)

so equation of chord T = S₁

$$hx + ky = h^2 + k^2$$

$$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k} \dots\dots(ii)$$

by (i) and (ii)

$$m = -\frac{h}{k} \text{ and } \sqrt{9m^2 - 16} = \frac{h^2 + k^2}{k}$$

$$9\frac{h^2}{k^2} - 16 = \frac{(h^2 + k^2)^2}{k^2}$$

$$\text{locus } 9x^2 - 16y^2 = (x^2 + y^2)^2$$

4. If a, b, c are three numbers where $b = a + c$ & S.D. of a + 2, b + 2, c + 2 is d then which of the following statement is true.

(1) $a^2 = 3b^2 + 3c^2 - d^2$

(2) $a^2 = b^2 + c^2 - d^2$

(3) $b^2 = 3b^2 + 3c^2 + 9d^2$

(4) $9d^2 = 3a^2 + 3c^2 - b^2$

Ans. (4)

Sol. for a, b, c

$$\text{mean} = \bar{x} = \frac{a+b+c}{3}$$

$$\bar{x} = \frac{2b}{3}$$

S.D. of a, b, c = d

$$d^2 = \frac{a^2 + b^2 + c^2}{3} - \frac{4b^2}{9}$$

$$b^2 = 3a^2 + 3c^2 - 9d^2$$

5. If $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$ then $(a + b + c)$ equal to

Ans. 4

Sol.
$$\lim_{x \rightarrow 0} \frac{\left\{ a \left(1 + x + \frac{x^2}{2!} + \dots \right) - b \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots \right) + c \left(1 - x + \frac{x^2}{2!} \dots \right) \right\}}{x \left(x - \frac{x^3}{3!} + \dots \right)} = 2$$

$$\therefore \lim_{x \rightarrow 0} \frac{(a - b + c) + x(a - c) + x^2 \left(\frac{a}{2} + \frac{b}{2} + \frac{c}{2} \right) + \dots}{x^2 \left(1 - \frac{x^2}{6} \dots \right)} = 2$$

$$\therefore a - b + c = 0$$

$$a - c = 0$$

$$\frac{a}{2} + \frac{b}{2} + \frac{c}{2} = 2$$

$$a + b + c = 4$$

6. which of the following is tautology ?

(1) $(p \wedge q) \rightarrow (p \rightarrow q)$

(2) $(p \wedge q) \vee (p \rightarrow q)$

(3) $(p \wedge q) \wedge (p \rightarrow q)$

(4) $(p \vee q) \rightarrow (p \rightarrow q)$

Ans. (1)

Sol.

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T	T
F	T	F	T	T	T
T	F	F	T	F	T
F	F	F	F	T	T

7. Number of irrational term in $(5^{1/4} + 3^{1/8})^{60}$ is n then which of the following is a factor of $(n - 1)$

(1) 26

(2) 5

(3) 7

(4) 27

Ans. (1)

Sol. $T_{r+1} = {}^{60}C_r (5^{1/4})^{60-r} (3^{1/8})^r$

rational if $\frac{60-r}{4}, \frac{r}{8}$ both are whole numbers, $r \in \{0, 1, 2, \dots, 60\}$

$$\frac{60-r}{4} \in W \Rightarrow r \in \{0, 4, 8, \dots, 60\}$$

$$\text{and } \frac{r}{8} \in W \Rightarrow r \in \{0, 8, 16, \dots, 56\}$$

$$\therefore \text{Common terms } r \in \{0, 8, 16, \dots, 56\}$$

So 8 terms are rational

Then Irrational terms = $61 - 8 = 53 = n$

$$\therefore n - 1 = 52 = 13 \times 2^2$$

factors 1, 2, 4, 13, 26, 52

8. If $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

Find 'n'

(1) 10

(2) 11

(3) 12

(4) 13

Ans. (3)

Sol. $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$

$$\sin x \cos x = \frac{1}{10} \quad \dots(1)$$

$$\text{and } \log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$$

$$\Rightarrow \sin x + \cos x = \left(\frac{n}{10}\right)^{\frac{1}{2}}$$

$$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{n}{10} \quad (\text{squaring})$$

$$\Rightarrow 1 + 2\left(\frac{1}{10}\right) = \frac{n}{10} \quad (\text{using Equation (1)})$$

$$\Rightarrow \frac{n}{10} = \frac{12}{10} \Rightarrow n = 12$$

9. If $f(x) + f(x+1) = 2$, $I_1 = \int_0^8 f(x) dx$ and $I_2 = \int_{-1}^3 f(x) dx$, then $I_1 + 2I_2$ is

Ans. 16

Sol. $f(x) + f(x+1) = 2 \quad \dots(i)$

$$x \rightarrow (x+1)$$

$$f(x+1) + f(x+2) = 2 \quad \dots(ii)$$

by (i) & (ii)

$$f(x) - f(x+2) = 0$$

$$f(x+2) = f(x)$$

$f(x)$ is period with $T = 2$

$$I_1 = \int_0^{2 \times 4} f(x) dx = 4 \int_0^2 f(x) dx$$

$$I_2 = \int_{-1}^3 f(x) dx = \int_0^4 f(x+1) dx = \int_0^4 (2 - f(x)) dx$$

$$I_2 = 8 - 2 \int_0^2 f(x) dx$$

$$I_1 + 2I_2 = 16$$

10. If $(|x| - 3)(|x + 4|) = 6$ then find the number of values of x ($x \in \mathbb{R}$ given)
 (1) 1 (2) 2 (3) 3 (4) 4

Ans. (2)

Sol. Case-1 $x \leq -4$

$$\begin{aligned} (-x - 3)(-x - 4) &= 6 \\ \Rightarrow (x + 3)(x + 4) &= 6 \\ \Rightarrow x^2 + 7x + 6 &= 0 \\ \Rightarrow x &= -1 \text{ or } -6 \end{aligned}$$

but $x \leq -4$

$$x = -6$$

Case-2 $x \in (-4, 0)$

$$\begin{aligned} (-x - 3)(x + 4) &= 6 \\ \Rightarrow -x^2 - 7x - 12 - 6 &= 0 \\ \Rightarrow x^2 + 7x + 18 &= 0 \end{aligned}$$

$D < 0$ No solution

Case-3 $x \geq 0$

$$\begin{aligned} (x - 3)(x + 4) &= 6 \\ \Rightarrow x^2 + x - 12 - 6 &= 0 \\ \Rightarrow x^2 + x - 18 &= 0 \end{aligned}$$

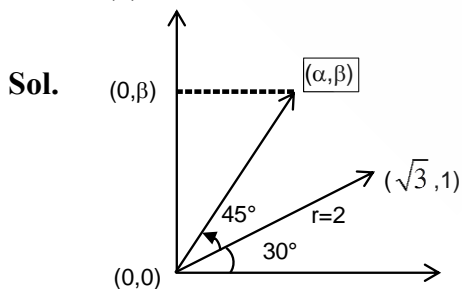
$$x = \frac{-1 \pm \sqrt{1 + 72}}{2}$$

$$\therefore x = \frac{\sqrt{73} - 1}{2} \text{ only}$$

11. If vector $(\sqrt{3} \hat{i} + \hat{j})$ is rotated 45° counter clockwise about origin then the new vector is $(\alpha \hat{i} + \beta \hat{j})$ then find the area of triangle whose co-ordinates are $(0, 0)$, $(0, \beta)$ and (α, β)

- (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) 2

Ans. (3)



$$(\alpha, \beta) \equiv (2 \cos 75^\circ, 2 \sin 75^\circ)$$

$$\text{Area} = \frac{1}{2} (2 \cos 75^\circ)(2 \sin 75^\circ)$$

$$= \sin(150^\circ) = \frac{1}{2} \text{ square unit}$$

12. Number of solution of equation $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in $[0, \pi]$ are

- (1) 2 (2) 3 (3) 4 (4) 5

Ans. (3)

Sol. $(81)^{\sin^2 x} + (81)^{1-\sin^2 x} = 30$

$$(81)^{\sin^2 x} + \frac{81}{(81)^{\sin^2 x}} = 30$$

$$t + \frac{81}{t} = 30 \quad \Rightarrow \quad t^2 + 81 = 30t$$

$$t^2 - 30t + 81 = 0$$

$$t^2 - 27t - 3 + 81 = 0$$

$$(t - 3)(t - 27) = 0$$

$$T = 3, 27$$

$$(81)^{\sin^2 x} = 3, 3^3$$

$$3^{4\sin^2 x} = 3^1, 3^3$$

$$4 \sin^2 x = 1, 3$$

$$\sin^2 x = \frac{1}{4}, \frac{3}{4}$$

in $[0, \pi]$ $\sin x > 0$

$$\sin x = \frac{1}{2}, \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$$

Number of solution = 4

13. If one card is missing out of 52 cards deck then out of remaining 51 cards two cards are drawn if both cards are found to be spade then the probability that missing card was NOT spade, is :

- (1) $\frac{21}{50}$ (2) $\frac{13}{50}$ (3) $\frac{39}{50}$ (4) $\frac{4}{5}$

Ans. (3)

Sol. $P\left(\bar{S}_{\text{missing}} \mid \text{both found spade}\right)$

$$\frac{P(\bar{S}_m \cap \text{BFS})}{P(\text{BFS})}$$

$$= \frac{\left(1 - \frac{13}{52}\right) \times \frac{13}{51} \times \frac{12}{50}}{\left(1 - \frac{13}{52}\right) \times \frac{13}{51} \times \frac{12}{50} + \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}}$$

$$= \frac{39}{50}$$

14. If $\frac{dy}{dx} + 2y \tan x = \sin x$, $y\left(\frac{\pi}{3}\right) = 0$, then maximum value of function

Ans. $\left(\frac{1}{8}\right)$

Sol. $\frac{dy}{dx} + 2 \tan x \cdot y = \sin x$

$$\text{I.F.} = e^{2 \int (\sec x)} = \sec^2 x$$

$$y \cdot \sec^2 x = \int \sin x \sec^2 x \, dx = \int \tan x \sec x \, dx + c$$

$$y \sec^2 x = \sec x + c$$

$$y = \cos x + c \cos^2 x$$

$$x = \frac{\pi}{3}, y = 0$$

$$\Rightarrow \frac{1}{2} + \frac{c}{4} \Rightarrow c = -2$$

$$\therefore y = \cos x - 2 \cos^2 x$$

$$y = -2 \left(\cos^2 x - \frac{1}{2} \cos x \right) = -2 \left(\left(\cos x - \frac{1}{4} \right)^2 - \frac{1}{16} \right)$$

$$y = \frac{1}{8} - 2 \left(\left(\cos x - \frac{1}{4} \right)^2 \right)$$

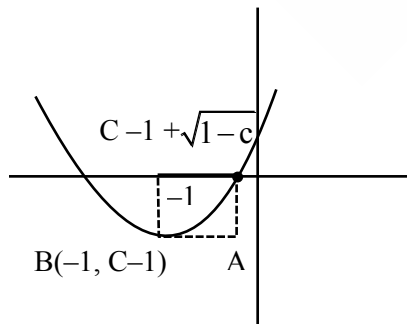
$$\therefore y_{\max} = \frac{1}{8}$$

15. If curve $y = f(x)$ satisfy the differential equation $\frac{dy}{dx} = 2(x+1)$ & area bounded by $y = f(x)$ & x-axis is $\frac{4\sqrt{8}}{3}$, then find $f(1)$

axis is $\frac{4\sqrt{8}}{3}$, then find $f(1)$

Ans. (2)

Sol. $y = x^2 + 2x + c$



$$\text{Area of rectangle (ABCD)} = |(c-1)(\sqrt{1-c})|$$

$$\text{Area of parabola and x-axis} = 2 \left(\frac{2}{3} ((1-c)^{3/2}) \right) = \frac{4\sqrt{8}}{3}$$

$$1 - c = 2 \Rightarrow c = -1$$

$$\text{Equation of } f(x) = x^2 + 2x - 1$$

$$f(1) = 1 + 2 - 1 = 2$$

16. If $y(x) = \int_0^x (2t^2 - 15t + 10) dt$,

Normal of above curve at P(a,b) is parallel to $3y + x = 5$ then find $|a + 6b|$ (given $a > 1$)

Ans. 406

Sol. $y'(x) = (2x^2 - 15x + 10)$

at point P

$$3 = (2a^2 - 15a + 10)$$

$$\Rightarrow 2a^2 - 15a + 7 = 0$$

$$\Rightarrow 2a^2 - 14a - a + 7 = 0$$

$$\Rightarrow 2a(a - 7) - 1(a - 7) = 0$$

$$a = \frac{1}{2} \text{ or } 7,$$

given $a > 1 \therefore a = 7$

also P lies on curve

$$\therefore b = \int_0^a (2t^2 - 15t + 10) dt$$

$$b = \int_0^7 (2t^2 - 15t + 10) dt$$

$$6b = -413$$

$$\therefore |a + 6b| = 406$$

17. If elements of 3×3 order matrix A are $\{0,1,2,3\}$ and trace of (AA^T) is 9 then total number of such matrices are.

Ans. 766

Sol. $AA^T = \begin{bmatrix} x & y & z \\ a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x & a & d \\ y & b & e \\ z & c & f \end{bmatrix}$

$$= \begin{bmatrix} x^2 + y^2 + z^2 & ax + by + cz & dx + ey + fz \\ ax + by + cz & a^2 + b^2 + c^2 & ad + be + cf \\ dx + ey + fz & ad + be + cf & d^2 + e^2 + f^2 \end{bmatrix}$$

$$\text{Tr}(AA^T) = x^2 + y^2 + z^2 + a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 9$$

all $\rightarrow 1$

1

one 3, rest = 0

$$\frac{9!}{8!} = 9$$

two 2, one 1 & rest 0

$$\frac{9!}{2!6!} = 63 \times 4 = 252$$

one 2, five 1, rest 0

$$\frac{9!}{5!3!} = 63 \times 8 = 504$$

$$= 766$$

18. If $f(x) = \log_2 \left(1 + \tan \frac{\pi x}{4} \right)$ find $\lim_{n \rightarrow \infty} \frac{2}{n} \left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f(1) \right)$

Ans. 1

Sol. $E = 2 \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(1 + \tan \frac{\pi x}{4} \right) dx \quad \dots(i)$$

replacing $x \rightarrow 1 - x$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(1 + \tan \frac{\pi}{4} (1 - x) \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(1 + \tan \left(\frac{\pi}{4} - \frac{\pi}{4} x \right) \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(1 + \frac{1 - \tan \frac{\pi}{4} x}{1 + \tan \frac{\pi}{4} x} \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left(\frac{2}{1 + \tan \frac{\pi x}{4}} \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \left(\ln 2 - \ln \left(1 + \tan \frac{\pi x}{4} \right) \right) dx \quad \dots(ii)$$

equation (i) + (ii)

$$E = 1$$

19. $\ln_{\frac{1}{\sqrt{2}}} \left(\frac{|z|+11}{(|z|-1)^2} \right) \leq 2$, find the maximum value of $|z|$.

Ans. (7)

Sol. $\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$

$$2|z| + 22 \geq (|z| - 1)^2$$

$$2|z| + 22 \geq |z|^2 - 2|z| + 1$$

$$|z|^2 - 4|z| - 21 \leq 0$$

$$(|z| - 7)(|z| + 3) \leq 0$$

$$\Rightarrow |z| \leq 7$$

$$\therefore |z|_{\max} = 7$$

20. Find value $\sum_{r=1}^{\infty} \tan^{-1} \left(\frac{6^r}{2^{(2r+1)} + 3^{(2r+1)}} \right)$

(1) $\cot^{-1} \frac{3}{2}$

(2) $\tan^{-1} \frac{3}{2}$

(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{4}$

Ans. (1)

Sol. $\sum_{r=1}^{\infty} \tan^{-1} \left(\frac{6^r(3-2)}{1 + \left(\frac{3}{2}\right)^{2r+1}} \right)$

$$\sum_{r=1}^{\infty} \tan^{-1} \left(\frac{2^r \cdot 3^{r+1} - 3^r 2^{r+1}}{2^{2r+1} + 3^{2r+1}} \right)$$

$$\sum_{r=1}^{\infty} \tan^{-1} \left(\frac{\left(\frac{3}{2}\right)^{r+1} - \left(\frac{3}{2}\right)^r}{1 + \left(\frac{3}{2}\right)^{r+1} \left(\frac{3}{2}\right)^r} \right) = \sum_{r=1}^{\infty} \left[\tan^{-1} \left(\frac{3}{2}\right)^{r+1} - \tan^{-1} \left(\frac{3}{2}\right)^r \right] = \frac{\pi}{2} - \tan^{-1} \frac{3}{2} = \cot^{-1} \frac{3}{2}$$

21. If $f(x) = \begin{cases} x+2 & x \leq 0 \\ x^2 & x > 0 \end{cases}$ and $g(x) = \begin{cases} 3x-2 & x \geq 1 \\ x^3 & x < 1 \end{cases}$ then number of points of non-differentiability of $f \circ g(x)$

(1) 0

(2) 1

(3) 2

(4) 3

Ans. (2)

Sol. $f \circ g(x) = \begin{cases} x^3 + 2 & x \leq 0 \\ x^6 & 0 \leq x \leq 1 \\ (3x-2)^2 & x \geq 1 \end{cases}$

$\therefore f \circ g(x)$ is discontinuous at $x = 0$ then non-differentiable at $x = 0$

at $x = 1$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{(3(1+h) - 2)^2 - 1}{h} = 6$$

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{(1-h)^6 - 1}{-h} = 6$$

Number of points of non-differentiability = 1

22. Let set $A = \{4, 8, 16, 32, 21, 26, 11, 15\}$. If four numbers taken from set A they form Ist four terms of AP & again four numbers taken from these they form Ist four terms of GP. If last number of these series is a four digit numbers then find number of common terms in these two series.

Ans. (3)

Sol. AP – 11, 16, 21, 26

GP – 4, 8, 16, 32

So common terms are 16, 256, 4096

23. If $f(x) = (4a - 3)(x + \ln 5) + 2(a - 7) \cot \frac{x}{2} \cdot \sin^2 \frac{x}{2}$, then the complete set of values of 'a' for which critical point of $f(x)$ exist is :

- (1) $[1, \infty)$ (2) $(-1, 0)$ (3) $\left[\frac{-4}{3}, 2\right]$ (4) $\left[-2, \frac{-4}{3}\right]$

Ans. (3)

Sol. $f(x) = (4a - 3)(x + \ln 5) + 2(a - 7) \left(\frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \cdot \sin^2 \frac{x}{2} \right)$

$$f(x) = (4a - 3)(x + \ln 5) + (a - 7) \sin x$$

$$f'(x) = (4a - 3) + (a - 7) \cos x = 0$$

$$\cos x = \frac{-(4a - 3)}{a - 7}$$

$$-1 \leq -\frac{4a - 3}{a - 7} \leq 1$$

$$-1 \leq \frac{4a - 3}{a - 7} \leq 1$$

$$\frac{4a - 3}{a - 7} - 1 \leq 0 \quad \text{and} \quad \frac{4a - 3}{a - 7} + 1 \geq 0$$

$$\Rightarrow \frac{-4}{3} \leq a \leq 2$$