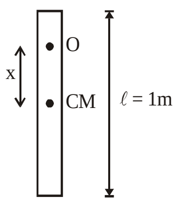
# HINTS & SOLUTIONS

## PART : I PHYSICS

### SECTION–I

**1.** **(1)**




$I_0 = \frac{m\ell^2}{12} + mx^2$

$T = 2\pi\sqrt{\frac{I_0}{mgx}} = 2(\text{given})$

$I_0 = mx$ [$\because \sqrt{g} = \pi$ ]

$\Rightarrow \frac{m}{12} + mx^2 = mx \Rightarrow 12x^2 - 12x + 1 = 0$

**2.** **(4)**

$Wd_{ext} = \Delta U = -\vec{M}.\vec{B}$

Case-1

W = – MB cos60° – (–MB cos 0°)

$\Rightarrow$ MB = 2W

Case-2

W' = –MB cos 120° – (–MB cos 60°)

W' = MB = 2W

**3.** **(2)**

$\rho \propto \frac{1}{T}(\text{given})$

ideal gas equation

$P = \frac{\rho}{M}RT$

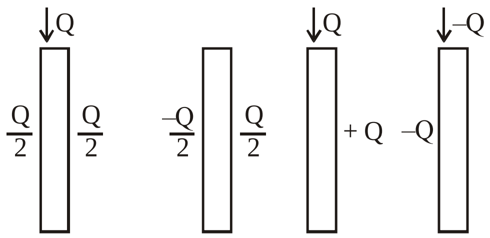
For constant pressure, $\rho \propto \frac{1}{T}$

$\Rightarrow$ Process is isobaric

so, $C_P = \frac{5R}{2}$ , for monoatomic gas.

**4.** **(3)**

Charge disribution



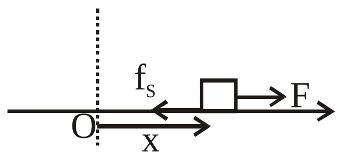

$E_0 = \frac{1}{2}\left(\frac{Q}{2}\right)^2 \times \frac{1}{C}$ $\quad E = \frac{1}{2}\frac{Q^2}{C}$

$E_0 = \frac{Q^2}{8C}$ $\quad E = 4E_0$

**5.** **(3)**

Green house effect involves "heating effect" which is brought about by "heat waves" of infrared rays.

**6.** **(3)**




$wd = \int (F - f_s)dx$

$= \int_{-L}^{L} \left(\frac{mg}{L}|x| - mg\frac{\mu_0|x|}{L}\right)dx$

$wd = \frac{mg}{L}(1-\mu_0)\int_{-L}^{L}|x|dx$

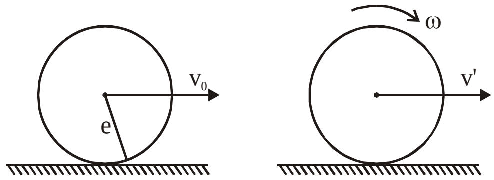
$= \frac{mg(1-\mu_0)}{L}\int_0^L 2x\,dx$

$wd = mg(1 - \mu_0)L$

$wd = \Delta KE$

$mg(1 - \mu_0)L = \frac{1}{2}mv_0^2$

**7. (3)**




$$v' = \frac{2v_0}{3}$$

$$I = mv_0 - mv'$$

$$I = \frac{mv_0}{3}$$

**8. (2)**

$$i = \frac{\varepsilon}{Res} = \frac{BDA}{t(Res.)} = \frac{B\times\pi\left(R^2 - \frac{R^2}{4}\right)}{t\times 2\pi R\lambda}$$

$$= \frac{B\times\pi 3R^2}{4\times t\times 2\pi\lambda}$$

$$i = \frac{3BR}{8t\lambda}$$

$$Q = it = \frac{3BR}{8\lambda}$$

**9. (2)**

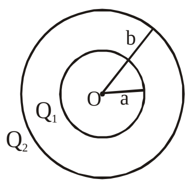
$g = g_0 (1 + \beta \sin^2\phi)$ (for small changes)

$\Delta g = \beta g_0\, 2\sin\phi \cos \beta\, \Delta\phi$

$\Delta g = bg_0 \sin 2\phi\Delta\phi$

$$\Delta\phi = \frac{\Delta g}{\beta g_0 \sin 2\phi}$$

**10. (1)**




Given $Q_1 + Q_2 = Q$

$$\frac{Q_1}{4\pi a^2} = \frac{Q_2}{4\pi b^2}$$

$$\frac{Q_1}{Q_2} = \frac{a^2}{b^2}$$

$$V_0 = \frac{kQ_1}{a} + \frac{kQ_2}{b}$$

$$= \frac{k}{a}\times\left(\frac{a^2}{a^2+b^2}\times Q\right) + \frac{k}{b}\left(\frac{b^2}{a^2+b^2}\times Q\right)$$

$$= \frac{kQ}{a^2+b^2}(a+b)$$

**11. (1)**

**12. (4)**

$$W_0 = \frac{hC}{\lambda_0}$$

$$\lambda_0 = \frac{hC}{W_0} = \frac{6.63\times10^{-34}\times3\times10^8}{6.63\times10^{-19}}$$

$$\lambda_0 = 3\times10^{-7}$$

$\lambda_0 = 300$ nm

For photoelectric emission

$\lambda < \lambda_0$

**13. (3)**

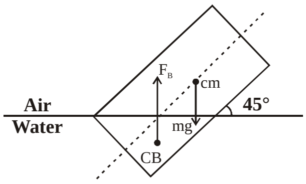
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$$

$$Y = \frac{T\ell}{A\Delta\ell} = \frac{T\ell}{A\ell\alpha t} = \frac{T}{A\alpha t}$$

$$T = YA\alpha t$$

$$v = \sqrt{\frac{YA\alpha t}{\rho A}} = \sqrt{\frac{Y\alpha t}{\rho}}$$

**14. (4)**




$$\tau = F_B \frac{a}{2\sqrt{2}}$$

$$\tau = \left[\rho_w \frac{a^2}{2} xg\right]\frac{a}{2\sqrt{2}}$$

$$w = \rho_B 2a^2 xg$$

$$\rho_w = 1g/cm^3, \rho_B = \frac{1}{4} g/cm^3$$

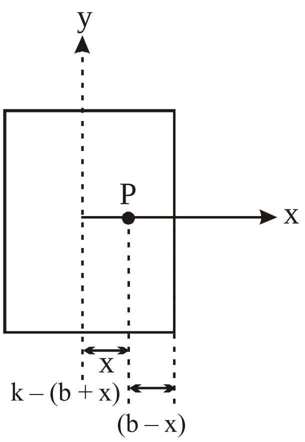
$$\tau = \frac{aW}{2\sqrt{2}}$$

**15. (2)**

**16. (2)**

**17. (1)**

**18. (3)**

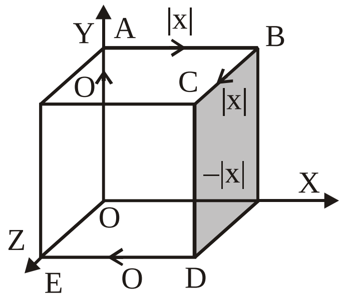



$$B_p = \mu_0 \frac{J(b+x)}{2} - \mu_0 J \frac{(b-x)}{2} + \frac{\mu_0 K}{2}$$

$$B_P = \mu_0 Jx + \mu_0 Jb$$

$$B_P = \mu_0 J(b+x)$$

**19. ( 2 )**




**20. (1)**

Resistance of Bulb: $R = \frac{(110)^2}{55} = 220\Omega$

Now in Series R-L Circuit rms voltage across R must be 110 V. Therefore rms current in the circuit will be 1/2 A.

$$Z = \frac{220}{1/2} = 2 \times 220 = 440\Omega$$

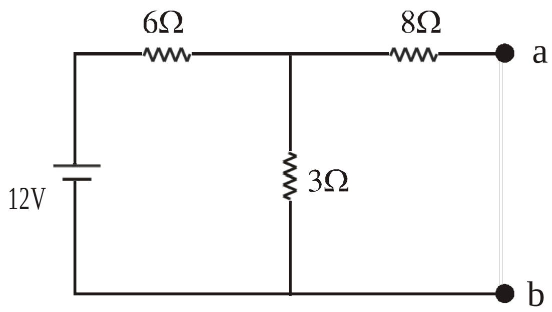
$$X_L{}^2 = Z^2 - R^2 = 440^2 - 220^2 = 145200$$

$$X_L = 381\Omega = \omega L = 2\pi \times 50 \times L$$

$$X_L \approx 1.2 H$$

## SECTION–II

**1. (00.40)**




$$P_{max} = \frac{(V_{ab})^2}{4R_{ab}}$$

$$V_{ab} = 4 volt$$

$$R_{ab} = 10\Omega$$

**2.** **(37)**

Consider this as horizontal projection from tower whose: Range = $\ell\sin\theta$ and Vertical height

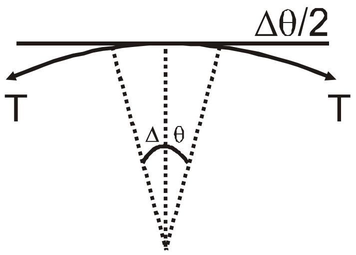
$= \ell(1-\cos\theta)$

$$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1-\cos\theta)}{10}} = \sqrt{\frac{(1-\cos\theta)}{5}}$$

$$R = u.T = 3.\sqrt{\frac{(1-\cos\theta)}{5}} = \sin\theta$$

On solving: $\theta = 37^o$

**3.** **(2)**



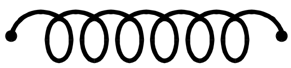

$$2T\sin\frac{\Delta\theta}{2} = d\, m\omega^2 r$$

$$2T\left(\frac{\Delta\theta}{2}\right) = \rho \times A \times r\Delta\theta \times \omega^2 \times r$$

$$\sigma = \frac{T}{A} = \rho r^2\omega^2$$

$$\therefore \quad \omega = \sqrt{\frac{\sigma}{\rho}} = 2 \text{ rad/s}$$

**4.** **(4)**



Using ; $V_A - V_B = RI + \frac{dI}{dt}L$

$140 = 5R + 10\,L$

$60 = 5R - 10\,L$

$\Rightarrow$ $L = 4H.$

**5.** **(8.00)**

Resonance frequency for both rod is same.

$f = \frac{nv}{2\ell_0}$, $\ell_0$ = length of rod, n = no. of loops

for ques.

$$\frac{1\times\sqrt{\frac{80}{\mu}}}{2\times2\ell} = \frac{4\times\sqrt{\frac{10}{\mu'}}}{2\times\ell} \Rightarrow \frac{80}{\mu} = 64\times\frac{10}{\mu'}$$

$$\frac{\mu'}{\mu} = 8$$

**6.** (220.00)

Heat gain = Heat loss

$(10 \times L_{ice} + 70 \times S_W \times 20)$

$= m \times \frac{S_w}{3} \times (50 - 20)$

$800 + 1400 = m \times 10$

$m = 220$ g

**7.** (0.05)

**8.** **(5)**

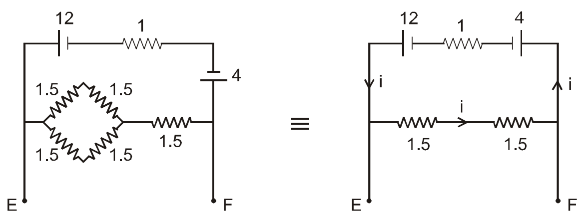
If $X_L = X_C$ current will be same,

So, $V_L = V_C$ ;

$\therefore\ V_L' = 1\times2\pi\times30\times\frac{1}{\pi} = 60\,\text{Volt}$

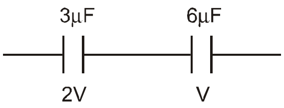
$V_R = 80\times1 = 80\,\text{volt}$

**9.** **(2)**




$$i = \frac{12-4}{1+1.5+1.5} = \frac{8}{4} = 2A$$

$V_{EF} = 3 \times 2 = 6V$




2V + V = 6

V = 2

$$V = \sqrt{V_L'^2 + V_R^2} = \sqrt{(80)^2 + (60)^2} = 100\,\text{Volt}$$

**10.** **(4)**

Before collision, linear momentum of the system is zero. Therefore, after collision momentum of B will be equal and opposite to momentum of C (since $P_A = 0$)

## PART : II CHEMISTRY

**1.** **(3)**

$H_4P_2O_7$

4(+1) + x + 7(–2) = 0

x = +5

$H_4P_2O_5$

4(+1) + x + 5(–2) = 0

x = +3

$H_4P_2O_6$

4(+1) + x + 6(–2) = 0

x = +4

**2.** **(2)**

$SO_2(g)$ is adsorbed to a larger extent than $H_2(g)$ because it is polar and have higher critical temperature than $H_2(g)$.

**3.** **(3)**

Correct order of ionisation enthalpy is

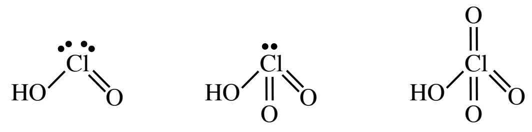
Al < Mg < S < P

In a period from left to right, atomic size decreases and Zeff increases, therefore I.E. increases. Due to stable configuration, Mg has higher I.E. than Al and P has higher than S.

**4.** **(4)**

According the Thomson model, all positive charge is spread over atom. Therefore heavy α-particles pass through the foil with decrease in speed and are deflected by small angles.

**5.** **(4)**



$HClO_2$ Chlorous acid — $HClO_3$ Chloric acid — $HClO_4$ Perchloric acid

**6.** **(2)**

During the extraction of copper from its sulphide ore, silica converts iron oxide into iron silicate slag.

**7.** **(4)**

Depression in freezing point ∝ i (Van't Hoff factor)

| | |
|---|---|
| For hydrazine ($NH_2–NH_2$) | i = 1 |
| For Glucose ($C_6H_{12}O_6$) | i = 1 |
| For Glycine ($NH_2–CH_2–COOH$) | i = 1 |
| For $KHSO_4$ ($K^+$, $HSO_4^-$) | i = 2 |

Hence, solution having $KHSO_4$ will exhibit the largest freezing point depression.

**8. (1)**

Cell constant: $m^{-1}$

Molar conductivity: $Scm^2mol^{-1}$

Conductivity: $\Omega^{-1}m^{-1}$

Degree of dissociation of electrolyte : dimensionless

**9. (1)**

Lithium salts are hydrated due to high hydration energy of $Li^+$

$Li^+$ due to smallest size in IA group has highest polarizing power.

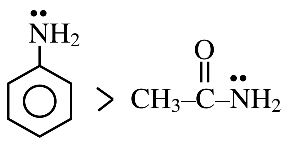
**10. (4)**

$Fe^{+2}(aq)$ (Green colour)

$Fe^{+3}(aq)$ (Yellow colour)

**11. (2)**

Aniline is more basic than acetamide because in acetamide, lone pair of nitrogen is atom present in conjugation with oxygen atom.




basic strength

**12. (3)**

(1) Eutrophication: The process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity.

(2) If the concentration of dissolved oxygen (DO) of water is below 6 ppm, the growth of fish gets inhibited.

(3) Eutrophication leads to decreases oxygen level in water.

(4) Eutrophication leads to anaerobic conditions due to decrease dissolved oxygen in water.

**13. (4)**

HI is very good reducing agent so we use oxidising reagent such as $HIO_3$ and $HNO_3$

**14. (3)**

Barfoed test is given by Glucose, Fructose, Maltose, Lactose, (for mono saccharides only)

**15. (4)**

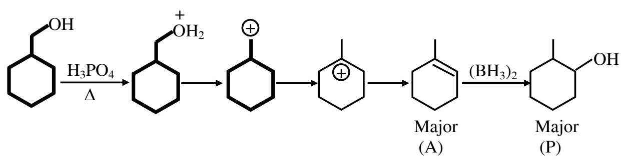
(a) Furacin is antiseptic which either kill or prevent the growth of microorganism.

(b) Arsphenamine is antibiotics which treat infections because of their low toxicity for humans and animals.

(c) Dimetone is synthetic antihistamines. They interfere with the natural action of histamine by competing with histamine for binding sites of receptor where histamine exerts its effect.

(d) Valium is transquilizers (neurologically active drug).

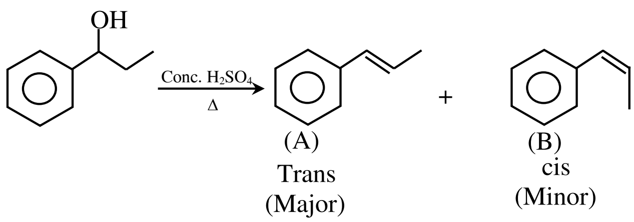
**16. (4)**



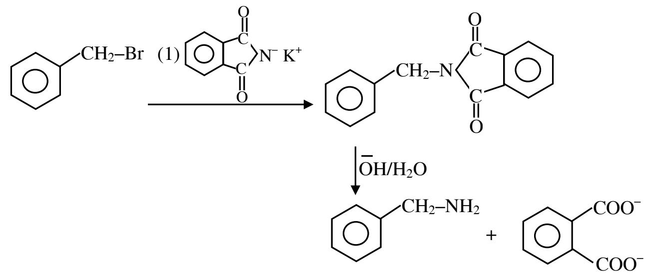

**17. (3)**

$$R\text{–}C\equiv N \xrightarrow[(2)\ H_2O]{(1)\ DIBAL\text{-}H} R-\underset{\underset{O}{||}}{C}-H$$

**18. (3)**



**19. (4)**




**20. (3)**

$CH_3-CH_2-CH_2-COO-CH_2-CH_3 \xrightarrow[H_3O^+]{LiAlH_4} CH_3-CH_2-CH_2-CH_2-OH + C_2H_5OH$

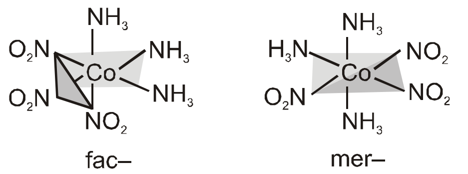
(A) (B)

(B) $\xrightarrow{\text{Oxidation}}$ $CH_3-CH_2-CH_2-COOH$

## SECTION-II

**1. (2)**

In $[Co(NO_2)_3C(NH_3)_3]$ no. of G.I. = 2 = x




In $[Cr(OX)_3]$ no. of G.I. = 0 = y (complex does not shows G.I.)

x + y = 2 + 0 = 2

**2. (16)**

$$A(s) \rightleftharpoons M(s) + \frac{1}{2}O_2(g)$$

$K_p = P_{O_2}^{1/2}$ or $P_{O_2} = K_p^2 = 4^2 = 16$

**3. (82)**

$H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

Mole = $\frac{0.2\times 400}{1000}$ = 0.08 ; Mole = $\frac{0.1\times 600}{1000}$ = 0.06 ; 0.03 mole ; 0.06 mole

Heat released = 0.06 × 57100 = 6 × 571 J

$q = mS\Delta T$

$6 \times 571 = 1000 \times 4.18 \times \Delta T$

$\Delta T = 81.9 \times 10^{-2}$ K

**4. (6)**

$3[Co(en)_2Cl_2]Cl + \underset{\text{excess}}{AgNO_3}$

$\longrightarrow 3[Co(en)_2Cl_2]NO_3 + 3AgCl$

Secondary valency of Co in $[Co(en)_2Cl_2]Cl$ is 6.

**5. (4)**

PbS, CuS, $As_2S_3$, CdS are soluble in 50% $HNO_3$. HgS, $Sb_2S_3$ are insoluble in 50% $HNO_3$

So Answer is 4.

**6. (1)**

Anions forms CCP or FCC $(A^-)$ = 4 $A^-$ per unit cell

Cations occupy all octahedral voids $(B^+)$ = 4 $B^+$ per unit cell

cell formula → $A_4B_4$

Empirical formula → AB

→ (x = 1)

**7. (47)**

$$\log k = \log A - \frac{E_a}{2.303RT}$$

Given $\log k = 20.35 - \frac{2.47\times 10^3}{T}$

$$\frac{E_a}{2.303R} = 2.47 \times 10^3$$

$$E_a = \frac{2.47\times 10^3 \times 2.303 \times 8.314}{1000} \text{KJmol}^{-1}$$

= 47.29 KJ/mol

**8. (0)**

$$O_2^{2-} \Rightarrow KK\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2Pz}^2\ \begin{matrix}\pi_{2Px}^2 & \pi^{*2}_{2Px}\\ \pi_{2Py}^2 & \pi^{*2}_{2Py}\end{matrix}$$

**9. (2)**

Number of atoms = $\frac{8}{23}\times 6.02\times 10^{23} = 2.09$

$\simeq 2$

**10. (5)**

Except Sn–$NH_4OH$, all will reduce nitrobenzene into aniline.

## PART : III MATHEMATICS

### SECTION–I

**1. (3)**

If, A + B = 45°

tan(A + B) = 1

$\Rightarrow \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow$ tan A + tan B = 1 – tan A tan B

$\Rightarrow$ (1 + tan A) (1 + tan B) = 2

LHS [(1 + tan2°)(1 + tan43°)] ... tan45°)

$1 + \tan\left(\dfrac{\pi}{4} - \theta\right) = 2 \,\big]\big]$

$1 + \tan\left(\dfrac{\pi}{4} - \theta\right) = 2 \,\big]\big]$

$= 2^{22}$ (1 + 1)

$= 2^{23}$

$= 2^{\lambda}$

then, λ = 23.

Hence the sum of digits of λ is 2 + 3 = 5

**2. (1)**

From given functional equation,

$2f(xy) = (f(x))^y + (f(y))^x, \forall x, y \in R$

putting y = 1, $2f(x) = f(x) + (f(1))^x$

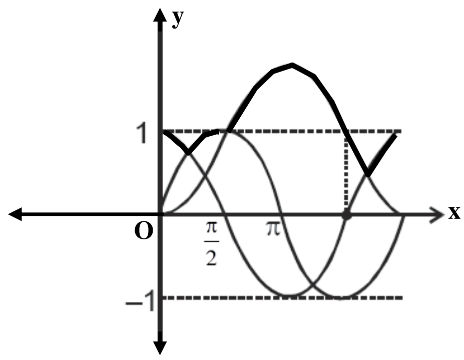
$2f(x) = f(x) + (f(1))^x$

$f(x) = 3^x$

$$\sum_{r=1}^{10} f(r) = \sum_{r=1}^{10} 3^r = \frac{3(3^{10} - 1)}{3 - 1} = \frac{3}{2}\left(3^{10} - 1\right)$$

**3. (1)**

The graph of f(x) = max. {sinx, cosx, 1 – cosx} is



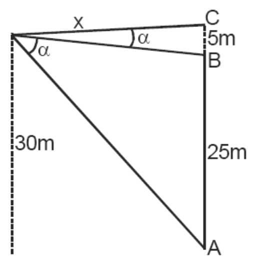

$\Rightarrow$ f(x) is not differentiable at $x = \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{5\pi}{4}$

$\Rightarrow$ f(x) is not differentiable at 3 points

**4. (2)**

We have $\tan\alpha = \dfrac{5}{x}$ and $\tan 2\alpha = \dfrac{30}{x}$

$\therefore \tan 2\alpha = \dfrac{30}{5\cot\alpha} \Rightarrow \tan 2\alpha = 6\tan\alpha$




Hence, $x = \dfrac{5}{\tan\alpha} \Rightarrow x = 5\sqrt{\dfrac{3}{2}}$m

**5. (3)**

Given expansion = $(1 + {}^mC_1x + {}^mC_2x^2 + .....)$ $(1 - {}^nC_1x + {}^nC_2x^2 - ......)$

Coefficient of $x^2 = -{}^nC_1 + {}^mC_1 = m - n = 3$

Coefficient of $x^2 = {}^nC_2 - {}^mC_1\,{}^mC_1 + {}^mC_2 =$

$\frac{n(n-1)}{2} - mn + \frac{m(m-1)}{2}$

$\Rightarrow \frac{n(n-1)}{2} - mn + \frac{m(m-1)}{2} = -6$

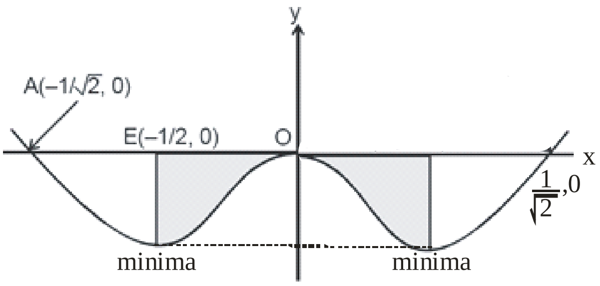
$n = m - 3$

$\frac{(m-3)(m-4)}{2} + \frac{m(m-1)}{2} - m(m-3) = -6$

$\Rightarrow -2m = -24 \Rightarrow m = 12$

**6. (2)**

The equation of the curve is




$y = 2x^4 - x^2 = (2x^2 - 1)x^2$

The curve is symmetrical about the y-axis.

Also, it is a polynomial of degree four having roots $0, 0, \pm\frac{1}{\sqrt{2}}$

x = 0 is repeated root. Hence, graph touches x – axis at (0, 0) and intersects the x-axis at

$A\left(\frac{1}{\sqrt{2}}, 0\right) \& B\left(\frac{1}{\sqrt{2}}, 0\right)$

$\Rightarrow \frac{dy}{dx} = 8x^3 - 2x = 2x(4x^2 - 1) = 0$

$x = 0, x = \pm\frac{1}{2}$

$\left(\frac{d^2y}{dx^2}\right) > 0$ at $x = \frac{1}{2}$ and at $x = \frac{-1}{2}$

So $x = \pm\frac{1}{3}$ are the points of local minima

Thus, the graph of the curve is shown in diagram.

Here, $y \le 0$, as x varies from $x = -\frac{1}{2}$ to $x = \frac{-1}{2}$

$\therefore$ The required area

= 2 Area OCDO

$= 2\left|\int_0^{\frac{1}{2}} y\,dx\right| = \left|\int_0^{\frac{1}{2}} (2x^4 - x^2)\,dx\right| = \frac{7}{120}$ sq.units

**7. (1)**

Let, p : Two triangles are identical

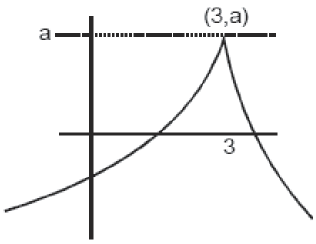
q : Two triangles are similar

Clearly, the given statement in symbolic form is $p \to q$.

Contrapositive of $p \to q$ is $\sim q \to \sim p$

i.e., If two triangles are not similar, then they are not identical.

**8. (3)**

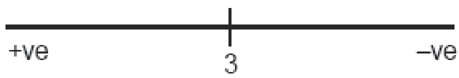



$\therefore f(x) = a - (x - 3)^{8/9}$

$\therefore f'(x) = 0 - \frac{8}{9}(x - 3)^{-1/9}$

At x = 3, f ' (x) is not defined.

hence sign scheme of f ' (x) is




$\therefore$ f(x) is maximum at x = 3

Hence maximum value of f(x) is equal to "a"

**9. (2)**

$x + a(2y + 1) = 0$

$x + b(3y + 1) = 0$

$x + a(4y + 1) = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4a & a \end{vmatrix} - 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 2a & a \end{vmatrix} - 0$$

$\Rightarrow 2a(b - a) = 0$

$2a = 0$ or $b = a$

Hence, locus of (a, b) is $y = x$ or $x = 0$

**10. (2)**

Let $y = \lim\limits_{t\to 0} \frac{2x}{\pi} \cot^{-1} \frac{x}{t^2}$

Case-I : when $x > 0$ then

$y = \frac{2x}{\pi} \lim\limits_{t\to 0} \cot^{-1} \frac{x}{t^2} = \frac{2x}{\pi} \times 0 = 0$

Case-II : when $x < 0$ then

$y = \frac{2x}{\pi} \lim\limits_{t\to 0} \cot^{-1} \frac{x}{t^2} = \frac{2x}{\pi} \times \pi = 2x$

$$f(x) = \begin{cases} \sin 0 & x > 0 \\ \sin 2x & x < 0 \end{cases}$$

Now, $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)dx$

$$= \int_{\frac{-\pi}{2}}^{0} \sin 2x dx + \int_{0}^{\frac{\pi}{2}} o dx = -\left(\frac{\cos 2x}{2}\right)_{-\frac{\pi}{2}}^{0} = -\frac{1}{2}(1 - (-1)) = -1$$

**11. (4)**

Given equation

$$4\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) - 57 = 0$$

Let, $x + \frac{1}{x} = y; x^2 + \frac{1}{x^2} = y^2 - 2$

$\Rightarrow 4y^2 + 16y - 65 = 0$

$\Rightarrow y = -\frac{13}{2}$ or $\frac{5}{2}$

When, $y = \frac{5}{2}$

$x + \frac{1}{x} = \frac{5}{2} \Rightarrow x = 2$ or $\frac{1}{2}$

When, $y = -\frac{13}{2}$

$\Rightarrow x + \frac{1}{x} = -13/2$

$\Rightarrow 2x^2 + 13x + 2 = 0$

Since x is rational, $x = 2$ or $\frac{1}{2}$

**12. (4)**

Given, x Ry $\Rightarrow$ x is relatively prime to y.

R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}

$\therefore$ Domain of R = {2, 3, 4, 5}

**13. (1)**

$\frac{\sum x_i}{20} = 10$ ....(i)

$\frac{\sum x_i}{20} - 100 = 4$ ....(ii)

$\sum x_i^2 = 104 \times 20 = 2080$

Actual mean = $\frac{200 - 9 + 11}{20} = \frac{202}{20}$

Variance = $\frac{2080 - 81 + 121}{20} = \left(\frac{202}{20}\right)^2$

$= \frac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99$

**14. (3)**

$$P(T_1) = \frac{20}{100}$$

$$P(T_2) = \frac{80}{100}$$

Let, $P = \left(\frac{D}{T_2}\right)$ = x (where, D represents defective units)

$$P = \left(\frac{D}{T_2}\right) = 10x$$

$$P(D) = \frac{7}{100}\text{(given)}$$

$$P(T_1)P\left(\frac{D}{T_1}\right) + P(T_2)P\left(\frac{D}{T_2}\right) = \frac{7}{100}$$

$$\frac{200}{100} \times 10x + \frac{80}{100} \times x = \frac{7}{100}$$

$$x = \frac{1}{40}$$

$$P\left(\frac{D}{T_2}\right) = \frac{1}{40}$$

$\Rightarrow P\left(\frac{\bar{D}}{T_2}\right) = \frac{39}{40}$ = Probability of not defective, given that it is produced in plant $T_2$

$$P\left(\frac{D}{T_1}\right) = \frac{10}{40} \Rightarrow P\left(\frac{\bar{D}}{T_1}\right) = \frac{30}{40}$$

Now using Bayes' theorem

Probability of computer from $T_2$ given that it is not defective :

$$P\left(\frac{T_2}{\bar{D}}\right) = \frac{\frac{80}{100} \times \frac{39}{40}}{\frac{20}{100} \times \frac{30}{40} + \frac{80}{100} \times \frac{39}{40}} = \frac{78}{93}$$

**15. (3)**

Given,

$|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$ and $\vec{a}.\vec{b} = 0 = \vec{b}.\vec{c} = \vec{c}.\vec{a}$

(as the three vectors are mutually perpendicular)

So,

$$\left[(\vec{a}+\vec{b}+\vec{c}) \times (\vec{b}-\vec{a})\right].\vec{c} = \left[\vec{a}\times\vec{b} - 0 + 0 - \vec{b}\times\vec{a} + \vec{c}\times\vec{b} - \vec{c}x \rightarrow\right].\vec{c}$$

$$= \left[2(\vec{a}\times\vec{b}).\vec{c} + (\vec{c}\times\vec{b}).\vec{c} - (\vec{c}\times\vec{a}).\vec{c}\right]$$

$$= 2(\vec{a}\times\vec{b}).\vec{c} + 0 - 0 = 2\left[\vec{a}\vec{b}\vec{c}\right] = 2.1.2.3 = 12$$

**16. (2)**

Let $2x + y = t \Rightarrow \frac{dy}{dx} + 2 = \frac{dt}{dx}$

$$\frac{dt}{dx} + xt = x^3t^3 \Rightarrow \frac{1}{t^3}\frac{dt}{dx} + \frac{1}{t^2}x = x^3$$

Let, $\frac{1}{t^2} = u \Rightarrow \frac{-2}{t^3}\frac{dt}{dx} = \frac{du}{dx}$

$$\frac{du}{dx} + (-2x)u = -2x^3$$

$$\text{I.F.} = e^{-\int 2x dx} = e^{-x^2} \Rightarrow u.e^{-x^2} = \int e^{-x^2}(-2x^3)dx$$

$$\frac{e^{-x^2}}{(2x+y)^2} = -2\int e^{-x^2}.x^3dx$$

$$\frac{e^{-x^2}}{(2x+y)^2} = \int e^{-x^2}.x^2(-2x)dx$$

Let, $-x^2 = v$

$$-2xdx = dv \Rightarrow \frac{e^{-x^2}}{(2x+y)^2} = -\int e^v v dv$$

$$\frac{e^{-x^2}}{(2x+y)^2} + v.e^v - e^v = C \Rightarrow$$

$$\frac{e^{-x^2}}{(2x+y)^2} - x^2e^{-x^2} - e^{-x^2} = C$$

$$\frac{1}{(2x+y)^2} = (x^2+1) + Ce^{x^2}$$

**17. (4)**

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$= (A + Bx)(x - A)^2$

Put $x = 0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A = -4$

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$= (Bx - 4)(x + 4)^2$

$$\begin{vmatrix} 1-\frac{4}{x} & 2 & 2 \\ 2 & 1-\frac{4}{x} & 2 \\ 2 & 2 & 1-\frac{4}{x} \end{vmatrix}$$

$= \left(B - \frac{4}{x}\right)\left(1 + \frac{4}{x}\right)^2$

Put $x \to \infty \Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

**18. (4)**

Two given lines intersect, if

$7\hat{i} + 10\hat{j} + 13\hat{k} + 5(2\hat{i} + 3\hat{j} + 4\hat{k})$

$= 3\hat{i} + 5\hat{j} + 7\hat{k} + t(\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow (7 + 2s)\hat{i} + (10 + 3s)\hat{j} + (13 + 4s)\hat{k}$

$= (3 + t)\hat{i} + (5 + 2t)\hat{j} + (7 + 3t)\hat{k}$

$\Rightarrow 7 + 2s = 3 + t$

$\Rightarrow 2s - t = -4$ ……(i)

$10 + 3s = 5 + 2t$

$\Rightarrow 3s - 2t = -5$ ……(ii)

And $13 + 4s = 7 + 3t$

$\Rightarrow 4s - 3t = -6$ ……(iii)

On solving equation (i) and (iii), we get equation

$s = -3, t = -2$

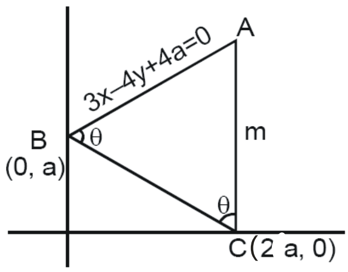
$\therefore$ Required point is

$7\hat{i} + 10\hat{j} + 13\hat{k}\left[2\hat{i} + 3\hat{j} + 4\hat{k}\right]$

$\hat{i} + \hat{j} + \hat{k}$

**19. (3)**

Let, $\angle ABC = \angle ACB = \theta$ and slope of AC = m




Slope of BC $= -\frac{1}{2}$

Slope of AB $= \frac{3}{4}$

Now, $\tan(\angle ABC) = \tan(\angle ACB)$

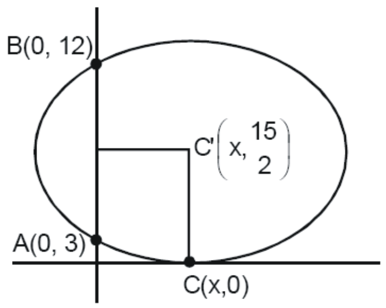
$$\Rightarrow \frac{\frac{3}{4} - \left(-\frac{1}{2}\right)}{1 + \left(\frac{3}{4}\right)\left(-\frac{1}{2}\right)} = \frac{-\frac{1}{2} - m}{1 + \left(-\frac{1}{2}m\right)}$$

$$\Rightarrow 2 = \frac{-\frac{1}{2} - m}{1 - \frac{m}{2}} \Rightarrow 2 - m = -\frac{1}{2} - m$$

$\Rightarrow$ m = not defined

$\Rightarrow$ equation of AC is $x = 2a$

**20. (4)**




$OA \times OB = OC^2 \Rightarrow 3 \times 12 = x^2 \Rightarrow x = 6$

Centre (C') = $\left(6, \frac{15}{2}\right)$, $\angle AC'B = 2\angle ACB = 2\theta$

$$\Rightarrow \cos 2\theta = \frac{\left(\frac{15}{2}\right)^2 + \left(\frac{15}{2}\right)^2 - 9^2}{2 \times \frac{15}{2} \times \frac{15}{2}}$$

$$= \frac{\frac{225}{2} - 81}{\frac{225}{2}} = \frac{225 - 162}{225} = \frac{63}{225} = \frac{7}{25}$$

## SECTION-II

**1. (12)**

$a^2 + b^2 = 50$

$a = 1 \Rightarrow b^2 = 49$

$\Rightarrow b = \pm 7$

a = 2,3,4,6 do not give integer value of b.

$a = 5 \Rightarrow b^2 = 25 \Rightarrow b = \pm 5$

$a = 7 \Rightarrow b^2 = 1$

$\Rightarrow b = \pm 1$

Similarly,

$a = -1 \Rightarrow b = \pm 7$

$a = -5 \Rightarrow b = \pm 5$

$a = -7 \Rightarrow b = \pm 1$

$\Rightarrow$ The required number of elements (a, b) = 12

**2. (16)**

$x^2 - ky + 32 = 0$

$\Rightarrow x^2 = k\left(y - \frac{32}{k}\right)$

Put, $x = X$, $y - \frac{32}{k} = Y$

The equation of directrix is $Y + \frac{k}{4} - 0$

i.e. $y - \frac{32}{k} + \frac{k}{4} - 0$

But, $y - 2 = 0$ is the directrix.

$\Rightarrow \frac{32}{k} - \frac{k}{4} - 2$

$\Rightarrow k^2 + 8k - 128 = 0$

$\Rightarrow k = -16$ or $k = 8$

For k = 8, the parabola is $x^2 = 8(y - 4)$ which does not intersect the circle.

For k = –16, the parabola is $x^2 = -16(y + 2)$ which intersects the circle at two real distinct points.

$\Rightarrow$ Absolute value of $k = |-16| = 16$

**3. (1)**

$|z_1| = |z_2| = |z_3| = 1$ (given)

Now $|z_1| = 1 \Rightarrow |z_1|^2 = 1$

$\Rightarrow z_1\overline{z_1} = 1 \Rightarrow \frac{1}{z_1} = \overline{z_1}$

Similarly, $\frac{1}{z_2} = \overline{z_2}$ & $\frac{1}{z_3} = \overline{z_3}$

Now $\left|\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right| = 1$

$\Rightarrow \left|\overline{Z_1 + Z_2 + Z_3}\right| = 1$ $(\because \overline{Z_1} + \overline{Z_2} = \overline{Z_1 + Z_2})$ and

$\left|\overline{Z_1 + Z_2 + Z_3}\right| = 1$ $(\because |\overline{Z_1}| = |Z|)$

$\Rightarrow \left|\overline{Z_1 + Z_2 + Z_3}\right| = 1$

**4. (37)**

Let $x = u^6$, $dx = 6u^5\,du$

$$\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = \int \frac{6u^2du}{u^3 + u^2} = 6\int \frac{u^3}{u+1}du = 6$$

$$\int\left(u^2 - u + 1 - \frac{1}{u+1}\right)du$$

$= 2u^3 - 3u^2 + 6u - 6\ln(u + 1) + e$

$\therefore a = 2, b = -3, c = 6, d = -6$

$\therefore 20a + b + c + d = 37$

**5. (0.5)**

$y = \tan^{-1}(\sec x - \tan x)$

$$y = \tan^{-1}\left(\frac{1 - \sin x}{\cos x}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}{\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)$$

$$\Rightarrow y = \frac{\pi}{4} - \frac{x}{2}$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{2} = 0.5$$

**6. (6)**

Let, x be the length of an edge, V be the volume and S be the surface area of the cube.

$$\frac{dv}{dt}=18, V=x^3 \Rightarrow \frac{dV}{dt}=18=3x^2\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt}=\frac{6}{x^2} \qquad .....(i)$$

$S = 6x^2$

$$\Rightarrow \frac{dS}{dt}=12x\frac{dx}{dt} \qquad ....[\text{From (i)}]$$

$$\Rightarrow \frac{dS}{dt}=12x\frac{6}{x^2}$$

$$\Rightarrow \frac{dS}{dt}=12\times\frac{6}{x}=6 \ [\because \text{Given } x = 12]$$

$$\therefore \frac{dS}{dt}=6cm^2/sec$$

**7. (5)**

Method I

We evaluate $A^2$ and $A^3$ and write the given equation as $AA^{-1} = I = \frac{1}{6}[A^3 + cA^2 + dA]$

Comparing the corresponding elements on both the sides, we get c = –6, d = 11

Method II

$$|A-xI| = 0 = \begin{vmatrix} 1-x & 0 & 0 \\ 0 & 1-x & 1 \\ 0 & -2 & 4-x \end{vmatrix} = 0$$

$\Rightarrow (1 - x) [(1 - x) (4 - x) + 2] = 0$

Hence characteristic equation is

$x^3 - 6x^2 + 11x - 6 = 0$

Then by Caley Hamilton theorem

$A^3 - 6A^2 + 11A - 6I = 0$

multiply by A–1 both the sides,

we get $\frac{1}{6}(A^2 - 6A + 11I) = A^{-1}$ .....(i)

given $A^{-1} = \frac{1}{6}(A^2 + cA + d)$ .....(ii)

then from equation (i) and (ii)

we get c = –6 d = 11

then c + d = 5

**8. (1)**

$\therefore$ Angles A, B, C are in arithmetic progression

and $\angle B = \frac{\pi}{4}$

then $A=\frac{\pi}{4}-\theta \ \ C=\frac{\pi}{4}+\theta$

Hence $\tan\left(\frac{\pi}{4}-\theta\right)\tan\frac{\pi}{4}\tan\left(\frac{\pi}{4}+\theta\right)$

$$=\frac{1-\tan\theta}{1+\tan\theta}.1.\frac{1+\tan\theta}{1-\tan\theta}=1$$

**9. (5.85)**

Continuity at $x = 2 \Rightarrow p(2) + q = 2^2 - 5(2) + 6$

$\Rightarrow q = -2p$

Continuity at

$x = 3 \Rightarrow a(9) + b(3) + 1 = 0$

Differentiable at

$x = 2 \Rightarrow p = 2(2) - 5 \Rightarrow p = -1$

Differentiable at $x = 3$

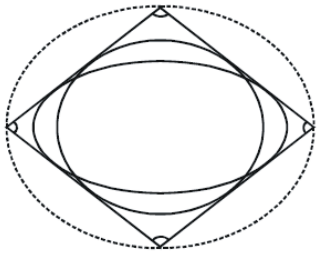
$\Rightarrow 2a(3) + b = 2(3) - 5 \Rightarrow 6a + b = 1$

$\therefore p = -1, q = 2, a = \frac{4}{9}, b = -\frac{5}{3}$

$|p| + |q| + \left|\frac{1}{a}\right| + \left|\frac{1}{b}\right| = 1 + 2 + \frac{9}{4} + \frac{3}{5}$

**10. (10)**

Director circle of $x^2 + y^2 = r^2$ and $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is same



$\Rightarrow x^2 + y^2 = 2r^2$ & $x^2 + y^2 = 16 + 9$ represents same circle

$\Rightarrow 2r^2 = 25 \Rightarrow r = \frac{5}{\sqrt{2}}$

side of square = $2r = 5\sqrt{2}$

diagonal of square = $\sqrt{2}$ (side of square) = $\sqrt{2}\ (5\sqrt{2}) = 10$