

# **JEE ADVANCED 2024**

# PAPER - 2



## **Duration : 3 Hours**

# **SUBJECT - PHYSICS**

### LEAGUE OF TOPPERS (Since 2020) TOP 100 AIRs IN JEE ADVANCED



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Ans.

### PHYSICS

#### **SECTION 1 (Maximum Marks: 12)**

- This section contains FOUR (04) questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated <u>according to the following marking scheme</u>: Full Marks : +3 If ONLY the correct option is chosen; Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered); Negative Marks : -1 In all other cases.
- 1. A region in the form of an equilateral triangle (in x-y plane) of height L has a uniform magnetic field  $\vec{B}$  pointing in the +z-direction. A conducting loop PQR, in the form of an equilateral triangle of the same height L, is placed in the x-y plane with its vertex P at x = 0 in the orientation shown in the figure. At t = 0, the loop starts entering the region of the magnetic field with a uniform velocity v along the +x-direction. The plane of the loop and its orientation remain unchanged throughout its motion.



Which of the following graph best depicts the variation of the induced emf (E) in the loop as a function of the distance (x) starting from x = 0?





for t = 0 to  $t = \frac{L}{V}$ Sol.



$$\ell = \left(\frac{\mathrm{vt}}{2}\tan 30^\circ\right)2 = \frac{\mathrm{vt}}{\sqrt{3}}$$

$$\phi = \left(\frac{1}{2}\frac{\mathrm{vt}}{2} \times \frac{\mathrm{vt}}{\sqrt{3}}\right) 2 = \frac{\mathrm{v}^2 \mathrm{t}^2}{2\sqrt{3}}$$

$$e.m.f = \frac{df}{dt} = -\frac{v^2 t}{2\sqrt{3}}$$

This function remain for t = 0 to  $t = \frac{L}{v}$ 

$$\phi = \left(\frac{1}{2}\frac{vt}{2} \times \frac{vt}{\sqrt{3}}\right)2 = \frac{v^2t^2}{2\sqrt{3}}$$
  
e.m.f =  $\frac{df}{dt} = -\frac{v^2t}{2\sqrt{3}}$   
This function remain for t = 0 to t =  $\frac{L}{v}$   
for t =  $\frac{L}{v}$  to t =  $\frac{2L}{v}$   
 $\frac{\sqrt{3}\ell}{4} - \frac{vt_1}{2}$   $\frac{\ell_2}{\sqrt{\sqrt{3}\ell}}$   $\frac{\sqrt{3}\ell}{2} - vt_1$ 

$$\ell_1 = 2\mathbf{v}\mathbf{t}_1 \tan 30^\circ = \frac{2\mathbf{v}\mathbf{t}_1}{\sqrt{3}}$$

$$\ell_2 = \left(\frac{\sqrt{3\ell}}{4} - \frac{\mathrm{vt}_1}{2} + \mathrm{vt}_1\right) \tan 30^\circ \times 2$$

$$\ell_2 = \frac{\ell}{2} + \frac{\mathbf{v}\mathbf{t}_1}{\sqrt{3}}$$



Shaded Area

$$\begin{split} &A\left[\frac{1}{2}\left(\frac{\sqrt{3\ell}}{4} + \frac{vt_1}{2}\right)\left(\frac{\ell}{2} + \frac{vt_1}{\sqrt{3}}\right) - \frac{1}{2}(vt_1)\left(\frac{2vt_1}{\sqrt{3}}\right)\right]2\\ &A = \frac{\sqrt{3\ell^2}}{8} + \frac{vt_1\ell}{2} - \frac{\sqrt{3}v^2t_1^2}{2}\\ &e.m.f. = -\frac{d\varphi}{dt} = -\frac{BdA}{dt} = B\left[\sqrt{3}v^2t_1 - \frac{v\ell}{2}\right]\\ &e.m.f. = 0 \Rightarrow t_1 = \frac{\ell}{2\sqrt{3}v} \rightarrow \text{ total time } t = \frac{\ell}{v} + \frac{\ell}{2\sqrt{3}v}\\ ∧ \text{ for } t_1 < \frac{\ell}{2\sqrt{3}v} \text{ e.m.f.} = -\text{ive and for } t > \frac{\ell}{2\sqrt{3}v} \text{ e.m.f.} = +\text{i} \end{split}$$

2. A particle of mass m is under the influence of the gravitational field of a body of mass M (>> m). The particle is moving in a circular orbit of radius  $r_0$  with time period  $T_0$  around the mass M. Then, the particle is subjected to an additional central force, corresponding to the potential energy  $V_c(r) = m\alpha/r^3$ , where  $\alpha$  is a positive constant of suitable dimensions and r is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius  $r_0$  in the combined gravitational potential due to M and  $V_c(r)$ , but with a new time period  $T_1$ , then  $(T_1^2 - T_0^2)/T_1^2$  is given by [G is the gravitational constant.]

(A) 
$$\frac{3\alpha}{GMr_0^2}$$
 (B)  $\frac{\alpha}{2GMr_0^2}$  (C)  $\frac{\alpha}{GMr_0^2}$  (D)  $\frac{2\alpha}{GMr_0^2}$ 

Ans. (A)

**Sol.** initially 
$$T_0^2 = \frac{4\pi^2 r_0^3}{GM}$$

new force 
$$f = -\frac{du}{dr} = \frac{3m\alpha}{r^4}$$

Centripetal force 
$$\frac{3M\alpha}{r_0^4} + \frac{GMm}{r_0^2} = \frac{mv^2}{r_0}$$



Time period 
$$T_1 = \frac{2\pi r_0}{v} \Rightarrow T_1^2 = \frac{4\pi^2 r_0^2}{v_0^2} = \frac{4\pi^2 v_0^2 \cdot r_0^3}{GMr_0^2}$$

$$\frac{T_1^2 - T_0^2}{T_1^2} = 1 - \frac{T_0^2}{T_1^2} = 1 - \frac{\frac{4\pi^2 r_0^3}{GM}}{\frac{r\pi^2 r_0^5}{3\alpha + GM r_0^2}} = 1 - \frac{3\alpha + GM r_0^2}{GM r_0^2}$$

$$=\frac{GMr_{0}^{2}-3\alpha-GMr_{0}^{2}}{GMr_{0}^{2}}=\frac{-3\alpha}{GMr_{0}^{2}}$$

A metal target with atomic number Z = 46 is bombarded with a high energy electron beam. The 3. emission of X-rays from the target is analyzed. The ratio r of the wavelengths of the  $K_{\alpha}$ -line and the cut-off is found to be r = 2. If the same electron beam bombards another metal target with Z = 41, the value of r will be

(A) 2.53 (B) 1.27 (C) 2.24 (D) 1.58  
(A)  

$$\sqrt{\frac{C}{\lambda_k}} = a(z-b) \Rightarrow \lambda_k = \frac{C}{a^2(z-b)^2}$$
also  $\lambda_{cut} = \frac{hc}{eV}$ 

Ans. (A)

**Sol.** 
$$\sqrt{\frac{C}{\lambda_k}} = a(z-b) \Rightarrow \lambda_k = \frac{C}{a^2(z-b)^2}$$

also  $\lambda_{\text{cut}} = \frac{\text{hc}}{\text{eV}}$ 

Given 
$$r = \frac{\lambda_k}{\lambda_{cut}} = \frac{eV}{ha^2(z-b)^2} \Rightarrow r \alpha \frac{1}{(z-b)^2}$$

$$\frac{\mathbf{r}_2}{\mathbf{r}_1} = \left(\frac{\mathbf{z}_1 - \mathbf{1}}{\mathbf{z}_2 - \mathbf{1}}\right)^2 = \left(\frac{46 - \mathbf{1}}{41 - 1}\right)^2 = \frac{81}{64}$$

$$\Rightarrow r_2 = \frac{81}{64} \times 2 = \frac{81}{32} \approx 2.53$$



4. A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass m and radius r and it is in a uniform vertical magnetic field  $B_0$ , as shown in the figure. Initially, it hangs vertically downwards, because of acceleration due to gravity g, on two conducting supports at P and Q. When a current I is passed through the loop, the loop turns about the line PQ by an angle  $\theta$  given by





Ans.

Sol.

case (i) Let  $\theta$  is upto equilibrium

- $\Rightarrow \tau_{\text{magnetic}} = \tau_{\text{mg}}$
- $B_0i(\pi r^2) \sin (90 \theta) = (mg \sin \theta)r$

$$\tan\theta = \frac{\pi r B_0 i}{mg}$$

case (ii) Let  $\theta$  is maximum angle rotate by loop



 $\Rightarrow$  K.E<sub>i</sub> + U<sub>i</sub> = K.E<sub>f</sub> + U<sub>f</sub>

$$\Rightarrow 0 + 0 + (-MB_0 \cos 90^\circ) = 0 + mgr(1 - \cos\theta) - MB_0 \cos (90 - \theta)$$

$$\Rightarrow \operatorname{mgr}\left(2\sin^2\frac{\theta}{2}\right) = \operatorname{B}_0 i\pi r^2\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$$

 $\Rightarrow \tan \frac{\theta}{2} = \frac{\pi r B_0 i}{mg}$  (This is not any option)

#### **SECTION 2 (Maximum Marks: 12)**

- This section contains **THREE (03)** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

- Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
- Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
- Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If unanswered;

Negative Marks : -2 In all other cases.

• For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

choosing ONLY (B) will get +1 mark;

choosing ONLY (D) will get +1 mark;

choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get -2 marks.



5. A small electric dipole  $\vec{p}_0$ , having a moment of inertia I about its center, is kept at a distance r from the center of a spherical shell of radius R. The surface charge density  $\sigma$  is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle  $\theta$  as shown in the figure. While staying at a distance r, the dipole is free to rotate about its center.



If released from rest, then which of the following statement(s) is(are) correct?

 $[\varepsilon_0 \text{ is the permittivity of free space.}]$ 

(A) The dipole will undergo small oscillations at any finite value of r.

- (B) The dipole will undergo small oscillations at any finite value of r > R.
- (C) The dipole will undergo small oscillations with an angular frequency of  $\sqrt{\frac{2\sigma p_0}{\epsilon_0 I}}$  at r = 2R
- (D) The dipole will undergo small oscillations with an angular frequency of  $\sqrt{\frac{\sigma p_0}{100 c}}$  at

r = 10R

#### Ans. (B,D)

**Sol.** Electric field inside shell is zero so if dipole is kept inside shell than it will not experience any force or torque

for r > R Torque on dipole =  $P_0\left(\frac{kq}{r^2}\right)\sin\theta = I\alpha$ 

$$\alpha = \left(\frac{P_0}{I} \cdot \frac{1}{4\pi \in_0} \cdot \frac{\sigma \cdot 4\pi R^2}{r^2}\right) \sin \theta \approx \left(\frac{P_0 \sigma R^2}{I \in_0 r^2}\right) \theta$$

$$\omega = \sqrt{\frac{P_0 \sigma R^2}{I \in_0 r^2}} \text{ for } r = 2R \ \omega = \sqrt{\frac{P_0 \sigma}{14I \in_0}}$$

and for 
$$r = 10R$$
  $\omega = \sqrt{\frac{P_0 \sigma}{100I \epsilon_0}}$ 



6. A Table tennis ball has radius  $(3/2) \times 10^{-2}$  m and mass  $(22/7) \times 10^{-3}$  kg. It is slowly pushed down into a swimming pool to a depth of d = 0.7 m below the water surface and then released from rest. It emerges from the water surface at speed v, without getting wet. and rises up to a height H. Which of the following option(s) is(are) correct?

[Given:  $\pi = 22/7$ , g = 10 m s<sup>-2</sup>, density of water = 1 × 10<sup>3</sup> kg m<sup>-3</sup>, viscosity of water = 1 × 10<sup>-3</sup> Pa-s.]

- (A) The work done in pushing the ball to the depth d is 0.077 J.
- (B) If we neglect the viscous force in water, then the speed v = 7 m/s.
- (C) If we neglect the viscous force in water, then the height H = 1.4 m.
- (D) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is 500/9.

Ans. (A,B)  
Sol. 
$$\oint_{mg}^{f_b} = \left(\frac{4}{3} \times \frac{22}{7} \times \frac{27}{8} \times 10^{-6}\right) 10^3 \times 10 = \frac{99}{7} \times 10^{-2} N$$

$$mg = \frac{22}{7} \times 10^{-3} \times 10 = \frac{22}{7} \times 10^{-2} N$$
(A)  $W = (f_b - mg)s = \left(\frac{77}{7} \times 10^{-2}\right)(.7) = 0.077J$ 
(B)  $a = \frac{f_b - mg}{m} = \frac{11 \times 10^{-2}}{\frac{22}{7} \times 10^{-3}} = \frac{70}{2} = 35 m / s^2$ 

$$v = \sqrt{2as} = \sqrt{2 \times 35 \times 0.7} = 7m / s$$
(D)  $f_1 = f_b - mg = 11 \times 10^{-2}$ 

$$f_2 = 6\pi rnv = 6 \times \frac{22}{7} \times \frac{3}{2} \times 10^{-2} \times 10^{-3} \times 7 = 18 \times 11 \times 10^{-5}$$

$$\frac{f_1}{f_2} = \frac{11 \times 10^{-2}}{18 \times 11 \times 10^{-5}} = \frac{1000}{18} = \frac{500}{9}$$

[when speed is 7 m/s, which is maximum but at that moment viscous force is zero so, option (D) is incorrect]



A positive, singly ionized atom of mass number A<sub>M</sub> is accelerated from rest by the voltage 7. 192 V. Thereafter, it enters a rectangular region of width w with magnetic field  $\vec{B}_0 = 0.1\hat{k}$  Tesla as shown in the figure. The ion finally hits a detector at the distance x below its starting trajectory. [Given: Mass of neutron/proton =  $(5/3) \times 10^{-27}$  kg, charge of the electron =  $1.6 \times 10^{-19}$  C]



Which of the following option(s) is(are) correct?

- (A) The value of x for  $H^+$  ion is 4 cm.
- (B) The value of x for an ion with  $A_M = 144$  is 48 cm.
- (C) For detecting ions with  $1 \le A_M \le 196$ , the minimum height  $\{x_1 x_0\}$  of the detector is 55 cm.
- Meashing (D) The minimum width w of the region of the magnetic field for detecting ions with  $A_M = 196$  is 56 cm.



R

R

Sol.

$$R = \frac{\sqrt{2m \text{ K.E}}}{qB}$$

$$R = \frac{\sqrt{2A_{M}.M_{P}.192e}}{eB}$$

$$R = \sqrt{A_{M}} \left( \frac{2 \times \frac{5}{3} \times 10^{-27} \times 192}{1.6 \times 10^{-19} \times 10^{-2}} \right) = \frac{\sqrt{A_{M}}}{50} = 2\sqrt{A_{M}} \text{ cm}$$



(A) for  $H^+ x 2R = 4\sqrt{1} = 4$  cm

(B) 
$$A_M = 144 \ x = 2R = 2R = \frac{2}{50}\sqrt{144} = 48 \text{ cm}$$

(C) 
$$X_{min} = 4\sqrt{1} = 4$$
 cm,  $X_{max} = 4\sqrt{196} = 56$  cm  $\Rightarrow x_1 - x_0 = 52$  cm

(D) W = R =  $2\sqrt{196}$  = 28 cm

#### **SECTION 3 (Maximum Marks: 24)**

- This section contains SIX (06) questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme: Full Marks : +4 If **ONLY** the correct integer is entered; Zero Marks : 0 In all other cases.
- 8. The dimensions of a cone are measured using a scale with a least count of 2 mm. The diameter of the base and the height are both measured to be 20.0 cm. The maximum percentage error in the determination of the volume is

Sol. 
$$V = \frac{1}{3}\pi r^{2}h = \frac{1}{12}\pi d^{2}h$$
$$\frac{\Delta V}{V} = \frac{2\Delta d}{d} + \frac{\Delta h}{h}$$
$$\frac{\Delta V}{V}\% = \left(\frac{2(2mm)}{20} + \frac{2mm}{20}\right) \times 100\% = 3\%$$

20cm

9. A ball is thrown from the location  $(x_0, y_0) = (0,0)$  of a horizontal playground with an initial speed  $v_0$  at an angle  $\theta_0$  from the +x-direction. The ball is to be hit by a stone, which is thrown at the same time from the location (x<sub>1</sub>, y<sub>1</sub> = (L, 0). The stone is thrown at an angle (180 –  $\theta_1$ ) from the + x-direction with a suitable initial speed. For a fixed  $v_0$ , when  $(\theta_0, \theta_1) = (45^\circ, 45^\circ)$ , the stone hits the ball after time T<sub>1</sub>, and when  $(\theta_0, \theta_1) = (60^\circ, 30^\circ)$ , it hits the ball after time T<sub>2</sub>. In such a case,  $(T_1/T_2)^2$  is



20cm )



for collision, both balls should have same vertical speed

$$\Rightarrow V_0 \sin\theta_0 = v_1 \sin\theta_1$$
  
time of collision  $T = \frac{L}{V_0 \cos\theta_0 + V_1 \cos\theta_0} = \frac{L}{V_0 \cos\theta_0 + \frac{V_0 \sin\theta_0 \cos\theta_1}{\sin\theta_1}}$ 
$$T = \frac{L}{V_0 \left(\cos\theta_0 + \sin\theta_0 \cot\theta_1\right)}$$
$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{\cos 60^\circ + \sin 60^\circ \cot 30^\circ}{\cos 45^\circ + \sin 45^\circ \cot 45^\circ}\right)^2 = \left(\frac{2}{\sqrt{2}}\right)^2 = 2$$

10. A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle  $\theta$  at P. as shown in the figure. When  $\theta = 30^{\circ}$ , then the electric flux through the curved surface of the cylinder is  $\Phi$ . If  $\theta = 60^{\circ}$ , then the electric flux through the curved surface becomes  $\Phi/\sqrt{n}$ , where the value of n is



#### Ans. (3)

**Sol.** flux through curved surface = flux through cylinder – flux through circular parts

$$\phi = \frac{q}{\epsilon_0} - 2 \times \frac{q}{2\epsilon_0} (1 - \cos\theta) = \frac{q}{\epsilon_0} \cos\theta$$
  
for  $\theta = 30^\circ$   $\phi = \frac{q}{\epsilon_0} \cos 30^\circ = \frac{\sqrt{3}q}{2\epsilon_0}$   
for  $\theta = 60^\circ$   $\phi' = \frac{q}{\epsilon_0} \cos 60^\circ = \frac{q}{2\epsilon_0} = \frac{\phi}{\sqrt{3}}$   
 $\Rightarrow n = 3$ 



11. Two equilateral-triangular prisms  $P_1$  and  $P_2$  are kept with their sides parallel to each other, in vacuum, as shown in the figure. A light ray enters prism  $P_1$  at an angle of incidence  $\theta$  such that the outgoing ray undergoes minimum deviation in prism  $P_2$ . If the respective refractive indices of



12. An infinitely long thin wire, having a uniform charge density per unit length of 5 nC/m, is passing through a spherical shell of radius 1 m, as shown in the figure. A 10 nC charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points P and R. in Volt, is \_\_\_\_\_.

[Given: in SI units  $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9$ , ln 2 = 0.7. Ignore the area pierced by the wire.]



#### Ans. 171

**Sol.**  $V_P - V_R = (V_P - V_R)_{Shell} + (V_P - V_R)_{Wire}$ 

$$V_{P} - V_{R} = \left(\frac{kq}{1} - \frac{kq}{2}\right) - 2k\lambda \ln\left(\frac{.5}{2}\right) = +\frac{kq}{2} + 4k\lambda \ln 2$$
$$V_{P} - V_{R} = 9 \times 10^{9} \left[\frac{10 \times 10^{-9}}{2} + 4 \times 5 \times 10^{-9} \times 0.7\right]$$
$$= 171$$

13. A spherical soap bubble inside an air chamber at pressure  $P_0 = 10^5$  Pa has a certain radius so that the excess pressure inside the bubble is  $\Delta P = 144$  Pa. Now, the chamber pressure is reduced to  $8P_0/27$  so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure  $\Delta P$  in both the cases to be much smaller than the chamber pressure. The new excess pressure  $\Delta P$  in Pa is\_\_\_\_\_.

Ans. 96



Potential

**Sol.** excess pressure  $\Delta P = 144 = \frac{4T}{R}$ 

As outside pressure decrease, volume of bubble increases



As process is isothermal

$$P_1 V_1 = P_2 V_2 \Longrightarrow \left( P_0 + \frac{4T}{R_1} \right) \cdot \frac{4}{3} \pi R_1^3 = \left( \frac{8P_0}{27} + \frac{4T}{R_2} \right) \frac{4}{3} \pi R_2^3$$

As Given  $\Delta_P \ll P_0$  We assume  $P_1 \approx P_0$  and  $P_2 \approx \frac{8P_0}{27}$ 

for calculation of R<sub>2</sub>

$$\Rightarrow P_0. R_1^3 = \frac{8P_0}{27} R_2^3 \Rightarrow R_2 = \frac{3R_1}{2}$$
$$\Delta P_2 = \frac{4T}{R_2} = \frac{4T}{3R_1/2} = \frac{2}{3} (\Delta P_1) = \frac{2}{3} \times 144 = 96$$

#### **SECTION 4 (Maximum Marks: 12)**

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are **TWO** (02) questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

**Full Marks** :+3 If ONLY the correct numerical value is entered in the designated place;

**Zero Marks** : 0 In all other cases.



#### **PARAGRAPH I**

In a Young's double slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8 mm. The distance between the slits at time t is given by  $d = (0.8 + 0.04 \sin \omega t)$  mm, where  $\omega = 0.08$  rad s<sup>-1</sup>. The distance of the screen from the slits is 1 m and the wavelength of the light used to illuminate the slits is 6000 Å. The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point O.



The 8<sup>th</sup> bright fringe above the point O oscillates with time between two extreme positions. The 14. s TE otentia separation between these two extreme positions, in micrometer (µm), is

Ans. 600 (600 to 602)

**Sol.** 
$$d_{\min} = 0.8 - 0.04 = 76$$
,  $d_{\max} = 0.8 + 0.04 = 0.84$ 

$$8_{\text{th}}$$
 bright friage y =  $\frac{8\lambda D}{1}$ 

$$y_{\text{max.}} - y_{\text{min.}} = 8\lambda D \left[ \frac{1}{d_{\text{min.}}} - \frac{1}{d \text{ max.}} \right]$$
  
= 8 × 6000 × 10<sup>-10</sup> × 1  $\left[ \frac{1}{.8 - .04} - \frac{1}{.8 + .04} \right] \times \frac{1}{10^{-3}}$   
= 48×10<sup>-4</sup>  $\left[ \frac{2 \times .04}{(.8)^2 - (.04)^2} \right] \approx \frac{8 \times 48 \times 10^{-6}}{(.4 \times 10^{-2})^2} = 600 \,\mu\text{m}$ 

Ans  $\rightarrow 600 \ \mu m$  Range (600–602)

On precise calculation without approximation  $(y_{max} - y_{min})$  comes near 601 µm. That's why range should be given for the answer.

= 0

The maximum speed in  $\mu$ m/s at which the 8<sup>th</sup> bright fringe will move is \_\_\_\_\_. 15.

Ans. 24

Sol. 
$$y = \frac{8\lambda D}{d} = \frac{8 \times 6 \times 10^{-7} \times 1}{(.8 \times 0.04 \sin \omega t) \times 10^{-3}} = \frac{48 \times 10^{-3}}{8 \times .4 \sin \omega t}$$
  
Speed  $V = \frac{dy}{dt} = 48 \times 10^{-3} \left[ \frac{.4\omega \cos \omega t}{(8 + .4 \sin \omega t)^2} \right]$   
clearly V is maximum when  $\cos \omega t = 1$  and  $\sin \omega t$   
 $V_{max.} = \frac{48 \times 10^{-3} \times .4 \times 8 \times 10^{-2}}{8 \times 8} = 24 \times 10^{-6} \text{ m/s}$ 

 $= 24 \ \mu m/s$ 



#### **PARAGRAPH II**

Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at  $x_0$ , are oscillating with amplitude a and angular frequency  $\omega$ . Thus, their positions at time t are given by  $x_1(t) = (x_0 + d) + a \sin \omega t$  and  $x_2(t) = (x_0 - d) - a \sin \omega t$ , respectively, where d > 2a. Particle 3 of mass m moves towards this system with speed  $u_0 = a\omega/2$ , and undergoes instantaneous elastic collision with particle 2, at time t<sub>0</sub>. Finally, particles 1 and 2 acquire a center of mass speed  $v_{cm}$  and oscillate with amplitude b and the same angular frequency  $\omega$ .



- 16. If the collision occurs at time  $t_0 = 0$ , the value of  $v_{cm}/(a\omega)$  will be
- 0.75 Ans.
- $v_1 = \frac{dx_1}{dt} = a\omega \cos \omega t$  and  $v_2 = \frac{dx_2}{dt} = -a\omega \cos \omega t$ Sol.

At 
$$t_0 = 0$$
 both balls are at their equilibrium position and  $v_1$  aw and  $v_2 = -a\omega$   

$$u_0 = \frac{a\omega}{0} + \frac{a$$

If the collision occurs at time  $t_0 = \pi/(2\omega)$ , then the value of  $4b^2/a^2$  will be \_\_\_\_\_. 17. Ans. 4.25

Sol. 
$$v_1 = \frac{dx_1}{dt} = a\omega \cos \omega t$$
 and  $v_2 = \frac{dx_2}{dt} = -a\omega \cos \omega t$   
At  $t_0 = \frac{\pi}{2\omega}$ ;  $v_1$ ,  $V_2$  are zero  $\Rightarrow$  both particles are at extreme  
 $v_1 = 0$   $v_2 = 0$   
before collision



v=0 $u_0 = a\omega/2$  $v_2 = 0$  $\overline{0000}$ after collision

Natural length of spring = 2d

At 
$$t_0 = \frac{\pi}{2\omega}$$
;  $X_1 - X_2 = (X_0 + d + a) - ((X_0 - d) - a) = 2d + 2a$ 

 $\Rightarrow$  elongation in spring is 2a

At maximum elongation velocity of both balls become same

$$\mathbf{v}_1^1 = \mathbf{v}_2^1 = \mathbf{v}_{\rm CM} = \frac{\mathbf{a}\boldsymbol{\omega}}{4}$$

by energy conservation

$$\frac{1}{2}m\left(\frac{a\omega}{2}\right)^{2}+0+\frac{1}{2}K(2a)^{2}=\frac{1}{2}2m\left(\frac{a\omega}{4}\right)^{2}+\frac{1}{2}KX^{2}$$

$$\frac{ma^{2}\omega^{2}}{8}-\frac{ma^{2}\omega^{2}}{16}=\frac{1}{2}K(X^{2}-4a^{2})=\frac{ma^{2}\omega^{2}}{16}$$

$$X^{2}-4a^{2}=\frac{ma^{2}\omega^{2}}{8K}=\frac{ma^{2}}{8K}\cdot\frac{2K}{m}=\frac{a^{2}}{4}$$
(for 2 blocks spring system  $\omega = \sqrt{\frac{K}{\mu}} = \sqrt{\frac{2K}{m}}$ )
$$X^{2}=\frac{17a^{2}}{4} \Rightarrow X = \sqrt{\frac{17a}{2}}$$

$$\Rightarrow Maximum and minimum elongation are  $\sqrt{\frac{17a}{2}}$ 

$$\Rightarrow Amplitude = \sqrt{\frac{17a}{4}} = b$$

$$\frac{4b^{2}}{a^{2}}=\frac{17a^{2}}{4a^{2}}=\frac{17}{4}=4.25$$
 Ans.$$

(for 2 blocks spring system  $\omega =$ 

$$X^2 = \frac{17a^2}{4} \Longrightarrow X = \sqrt{\frac{17a}{2}}$$

 $\Rightarrow$  Maximum and minimum elongation are

$$\Rightarrow \text{Amplitude} = \sqrt{\frac{17a}{4}} = b$$
$$\frac{4b^2}{a^2} = \frac{17a^2}{4a^2} = \frac{17}{4} = 4.25 \text{ Ans.}$$