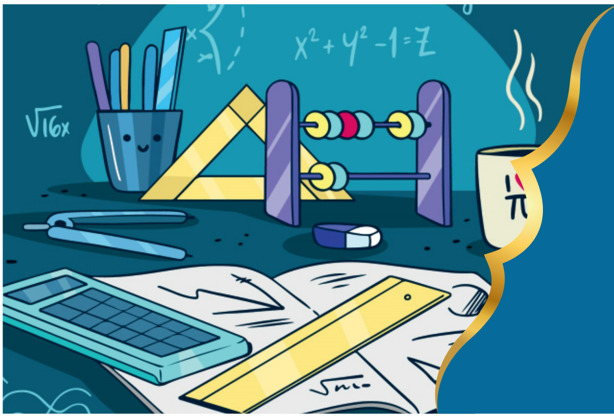
# JEE ADVANCED 2024

## PAPER - 2



## QUESTIONS & SOLUTIONS

26 MAY, 2024

02:30 PM to 05:30 PM

Duration : 3 Hours

## SUBJECT - MATHEMATICS



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# MATHEMATICS

## SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

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**1.** Considering only the principal values of the inverse trigonometric functions, the value of

$\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ is

(A) $\frac{7}{24}$ (B) $\frac{-7}{24}$ (C) $\frac{-5}{24}$ (D) $\frac{5}{24}$

**Ans.** **(B)**

**Sol.** $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$

$\tan\left(\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{4}{3}\right)\right)$

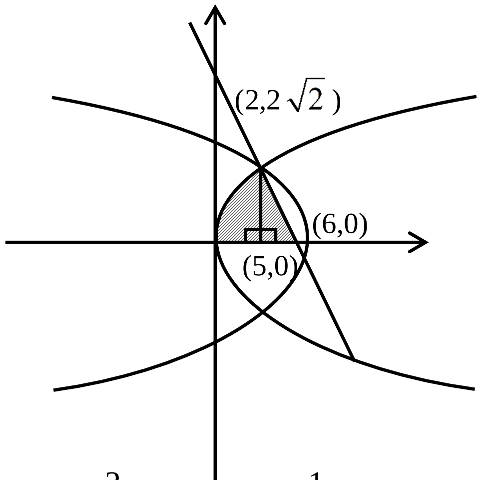
$\tan\left(\tan^{-1}\left(\frac{\frac{3}{4}-\frac{4}{3}}{1+\frac{3}{4}\times\frac{4}{3}}\right)\right)=\frac{-7}{24}$

**2.** Let $S=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: x\geq 0, y\geq 0, y^2\leq 4x, y^2\leq 12-2x \text{ and } 3y+\sqrt{8}x\leq 5\sqrt{8}\right\}$. If the area of the region S is $\alpha\sqrt{2}$, then $\alpha$ is equal to

(A) $\frac{17}{2}$ (B) $\frac{17}{3}$ (C) $\frac{17}{4}$ (D) $\frac{17}{5}$

**Ans.** **(B)**

**Sol.**




Area = $\frac{2}{3}\left(2\sqrt{2}\right)(2) + \frac{1}{2}\cdot(3)\left(2\sqrt{2}\right) = \frac{8}{3}\sqrt{2} + 3\sqrt{2} = \frac{17}{3}\sqrt{2} = \alpha\sqrt{2}$

$\Rightarrow \quad \alpha = \frac{17}{3}$

**3.** Let $k \in \mathbb{R}$. If $\lim_{x\to 0+}\left(\sin(\sin kx)+\cos x + x\right)^{\frac{2}{x}} = e^6$, then the value of k is

(A) 1 (B) 2 (C) 3 (D) 4

**Ans.** **(B)**

**Sol.** $\lim_{x\to 0^+}\left(\sin(\sin kx)+\cos x+x\right)^{\frac{2}{x}}$

$= e^{\lim_{x\to 0^+}\frac{\sin(\sin kx)+\cos x+x-1}{x}\times 2}$ ($1^\infty$ form)

$= e^{\lim_{x\to 0^+}\left[\left(\frac{\sin(\sin kx)}{\sin kx}\times\frac{\sin kx}{kx}\times k\right)+\frac{\left(1-\frac{x^2}{2}+x-1\right)}{x}\right]\times 2} = e^{\lim_{x\to 0^+}(k+1)2} = e^6$

$k + 1 = 3 \Rightarrow k = 2$

**4.** Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2\sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0 \end{cases}$$

Then which of the following statements is TRUE?

(A) f(x) = 0 has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$.

(B) f(x) = 0 has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$

(C) The set of solutions of f(x) = 0 in the interval $\left(0, \frac{1}{10^{10}}\right)$ is finite.

(D) f(x) = 0 has more then 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$.

**Ans.** **(D)**

**Sol.** $f(x) = x^2 \sin\left(\frac{\pi}{x^2}\right) = 0$

$\sin\left(\frac{\pi}{x^2}\right) = \sin \pi n$

$\frac{1}{x^2} = n \qquad n \in I^+$

(A) $x \in \left(\frac{1}{10^{10}}, \infty\right) \qquad \frac{1}{x} \in (0, 10^{10})$

$\frac{1}{x^2} \in (0, 10^{20}) \rightarrow$ finite solutions

(B) $\frac{1}{x} \in (0, \pi)$

$\frac{1}{x^2} \in (0, \pi^2) \rightarrow$ 9 solutions

(C) $\frac{1}{x} \in (10^{10}, \infty) \rightarrow$ infinite solutions

(D) $\frac{1}{x} \in (\pi, \pi^2)$

$\frac{1}{x^2} \in (\pi^2, \pi^4) \rightarrow (9.8, 96.1)$ approx

more than 25 solutions

**SECTION 2 (Maximum Marks: 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of option(s) will get –2 marks.

**5.** Let S be the set of all $(\alpha,\beta)\in\mathbb{R}\times\mathbb{R}$ such that

$$\lim_{x\to\infty}\frac{\sin\left(x^2\right)\left(\log_e x\right)^{\alpha}\sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}\left(\log_e(1+x)\right)^{\beta}}=0$$

Then which of the following is (are) correct?

(A) $(-1,3)\in S$ (B) $(-1,1)\in S$ (C) $(1,-1)\in S$ (D) $(1,-2)\in S$

**Ans.** **(BC)**

**Sol.** $$\lim_{x\to\infty}\frac{\sin(x^2)(\ell nx)^{\alpha}\sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta+2}(\ell n(1+x))^{\beta}\left(\frac{1}{x^2}\right)}$$

$$\lim_{x\to\infty}\frac{\sin(x^2)(\ell nx)^{\alpha-\beta}}{x^{\alpha\beta+2}}\underbrace{\left(\frac{\ell n x}{\ell n(1+x)}\right)^{\beta}}_{\text{as } x\to\infty,\text{ tends to }1}=0$$

Now, if $\alpha\beta + 2 > 0$, $\frac{(\ell n x)^{\alpha-\beta}}{x^{\alpha\beta+2}}\to 0$

$\therefore$ limit = 0 if $\alpha\beta + 2 > 0$

**6.** A straight line drawn from the point P(1,3,2), parallel to the line $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$, intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane $L_1$ intersects the plane $L_2 : 2x - y + z = -4$ at the point R. Then which of the following statements is (are) TRUE?

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are (1, 6, 3)

(C) The centroid of the triangle PQR is $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2}+\sqrt{6}+\sqrt{11}$

**Ans.** **(AC)**

**Sol.** Line passing through 'P' is $\frac{(x-1)}{1}=\frac{y-3}{2}=\frac{z-2}{1}=\lambda$

$C_1(\lambda + 1, 2\lambda + 3, \lambda + 2)$

put $C_1$ in L $\to (\lambda + 1) - (2\lambda + 3) + 3(\lambda + 2) = 6$

$\Rightarrow \quad \lambda = 1 \Rightarrow Q(2, 5, 3)$

Also, line through Q is $\frac{(x-2)}{1}=\frac{(y-5)}{-1}=\frac{(z-3)}{3}=k$

$C_2(k + 2, 5 - k, 3k + 3)$

put in $L_2 \to k = -1 \Rightarrow R(1, 6, 0)$

**7.** Let $A_1, B_1, C_1$ be three points in the xy-plane. Suppose that the lines $A_1C_1$ and $B_1C_1$ are tangents to the curve $y^2 = 8x$ at $A_1$ and $B_1$, respectively . if O=(0,0) and $C_1 = (-4,0)$, then which of the following statements is (are) TRUE?

(A) The length of the line segment $OA_1$ is $4\sqrt{3}$

(B) The length of the line segment $A_1B_1$ is 16

(C) The orthocentre of the triangle $A_1B_1C_1$ is (0,0)

(D) The orthocentre of the triangle $A_1B_1C_1$ is (1,0)

**Ans.** **(AC)**

**Sol.** $ty = x + at^2 \rightarrow (-4, 0) \Rightarrow 0 = -4 + 2t^2 \Rightarrow t = \pm\sqrt{2}$

$P(at^2, 2at) = P(2t^2, 4t)$

$A_1 (4, 4\sqrt{2}), B_1 (4, -4\sqrt{2}), C_1(-4, 0), O(0, 0)$

(A) $OA_1 = 4\sqrt{3}$

(B) $A_1B_1 = 8\sqrt{2}$

(C) Attitude trough $A_1$ is $\sqrt{2}x + y = 0 \Rightarrow$ orthocentre is (0, 0)

## SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

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**8.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and **g :** $\mathbb{R} \rightarrow (0, \infty)$ be a function such that $g(x + y) = g(x)\, g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right) g(0)$ is _____.

**Ans.** **(51)**

**Sol.** $f(0) = 0$

$f\left(\frac{-3}{5} + \frac{3}{5}\right) = f\left(\frac{-3}{5}\right) + f\left(\frac{3}{5}\right) \Rightarrow f\left(\frac{3}{5}\right) = -12$

$f(2x) = 2f(x)$

$f(3x) = f(2x) + f(x) = 3f(x)$

$f(4x) = 4f(x)$

$f(5x) = 5f(x)$

$f(nx) = nf(x)$

$x = \frac{3}{5}, n = 5 \quad f(3) = 5f\left(\frac{3}{5}\right) = -60$

$n = 1, x = 1 \quad f(3) = 3f(1) = -60 \Rightarrow f(1) = -20$

$n = 4, x = \frac{1}{4}$ $f(1) = 4f\left(\frac{1}{4}\right) \Rightarrow -20 = 4f\left(\frac{1}{4}\right) \Rightarrow f\left(\frac{1}{4}\right) = -5$

$g(x + y) = g(x)\, g(y)$

$g(0) = g^2(0) \Rightarrow g(0) = 1$

$g(nx) = g^n(x)$

$n = 6, x = -\frac{1}{3}$ $g(-2) = g^6\left(-\frac{1}{3}\right) = (2)^6 = 64$

$\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right) g(0)$

$(-5 + 64 - 8) \times 1 = 51$

**9.** A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue . Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For i = 1, 2, 3, let $W_i$, $G_i$, and $B_i$ denote the events that the ball drawn in the $i^{th}$ draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 | W_1 \cap G_2) = \frac{2}{9}$, then N equals_____.

**Ans.** **(11)**

**Sol.** $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$

$\frac{3}{N} \cdot \frac{6}{N-1} \cdot \frac{N-9}{N-2} = \frac{2}{5N} \Rightarrow N = 11 \text{ or } 37$

Also $\frac{P(B_3 \cap W_1 \cap G_2)}{P(W_1 \cap G_2)} = \frac{2}{9}$

$\Rightarrow \frac{\frac{2}{5N}}{\frac{3}{N} \cdot \frac{6}{N-1}} = \frac{2}{9} \Rightarrow N = 11$

**10.** Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)}.$$

Then the number of solutions of f(x) = 0 in $\mathbb{R}$ is ______.

**Ans.** **(1)**

**Sol.** $f(x) = 0 \Rightarrow x^{2023} + 2024x + 2025 = 0$

As LHS is strictly increasing, number of solutions is '1'

$(\sin x + 2 \neq 0 \text{ if } x \in R)$

**11.** Let $\vec{p} = 2\hat{j} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β, and γ, we have

$15\hat{i}+10\hat{j}+6\hat{k}=\alpha\left(2\vec{p}+\vec{q}\right)+\beta\left(\vec{p}-2\vec{q}\right)+\gamma\left(\vec{p}\times\vec{q}\right)$, then the value of $\gamma$ is _______.

**Ans.** **(2)**

**Sol.** $\vec{p} = 2\hat{i}+\hat{j}+3\hat{k}$

$\vec{q} = \hat{i}-\hat{j}+\hat{k}$

$$\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}\,(1+3) - \hat{j}\,(2-3) + \hat{k}\,(-2, -1) = 4\hat{i}+\hat{j}-3\hat{k}$$

Given equation $15\hat{i}+10\hat{j}+6\hat{k} = \alpha(2\vec{p}+\vec{q}) + \beta(\vec{p}-2\vec{q}) + \gamma(\vec{p}\times\vec{q})$

$(4\hat{i}+\hat{j}-3\hat{k})\ .\ (15\hat{i}+10\hat{j}+6\hat{k}) = \gamma\,|\vec{p}\times\vec{q}\,|^2$

$60 + 10 - 18 = \gamma(16 + 1 + 9)$

$52 = \gamma(26)$

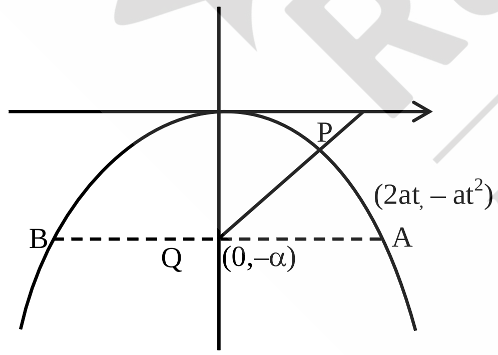
$\gamma = 2$

**12.** A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0,-\alpha)$ to the parabola $x^2 = -4ay$, where $a > 0$.

Let L be th line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length fo the line segment AB. IF r : s = 1 : 16, then the value of 24a is _____.

**Ans.** (12)

**Sol.** $m_N = \frac{1}{\sqrt{6}}$




$x^2 = -4ay$

$2x = -4ay'$

$y' = \frac{x}{-2a} = \frac{2at}{-2a} = -t$

$m_N = \frac{1}{t} = \frac{1}{\sqrt{6}} \Rightarrow t = \sqrt{6}$

$p\left(2\sqrt{6}a, -6a\right)$

equation of normal $(y + 6a) = \frac{1}{\sqrt{6}}(x - 2\sqrt{6}a)$

$(y + 6a) = \frac{1}{\sqrt{6}}(x - 2\sqrt{6}a)$

$y + 6a = -2a$

$y = -2a - 6a = -8a$

$\alpha = 8a$

Equation of line L

$y = -8a$

$x^2 = -4ay$

$x^2 = +32a^2 \Rightarrow x = \pm 4\sqrt{2}a$

$AB = 8\sqrt{2}a$

$S = 128a^2$

$r = 4a$

$\frac{r}{S} = \frac{4a}{128a^2} = \frac{1}{16} \Rightarrow a = \frac{64}{128} \Rightarrow a = \frac{1}{2}$

$24a = 12$

**13.** Let the function $f : [1, \infty) \to \mathbb{R}$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1}2, & \text{if } t = 2n-1, n \in \mathbb{N} \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1, n \in \mathbb{N} \end{cases}$$

Define $g(x) = \int_1^x f(t)\,dt, x \in (1, \infty)$. Let $\alpha$ denote the number of solutions of the equation $g(x) = 0$ in the interval (1,8] and $\beta = \lim_{x \to 1+} \frac{g(x)}{x-1}$. Then the value of $\alpha + \beta$ is equal to ____.

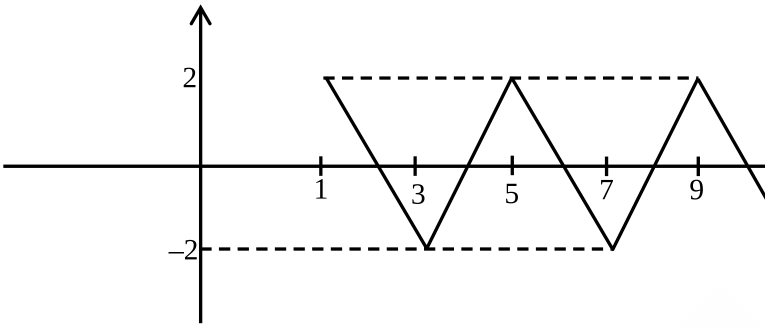
**Ans.** **(5)**

**Sol.** $$f(t) = \begin{cases} (-1)^{n+1}2 & ; t = 2n-1 \\ \frac{(2n+1-t)}{2}(-1)^{n+1}.2 + \frac{(t-2n+1)}{2}(-1)^{n+2}.2 & ; 2n-1 < t < 2n+1 \end{cases}$$

$$f(t) = \begin{cases} (-1)^{n+1}2 & ; t = 2n-1 \\ (-1)^{n+1}[2n+1-t-t+2n-1 & ; 2n-1 < t < 2n+1 \end{cases}$$

$$f(t) = \begin{cases} (-1)^{n+1}2 & ; t = 2n-1 \\ (-1)^{n+1}2(2n-t) & ; 2n-1 < t < 2n+1 \end{cases}$$

Graph of f(t)




$g(x) = \int_1^x f(t)\,dt$

Number of solution of equation g(x) = 0 in (1, 8] is 3

$\alpha = 3$

$$\beta = \lim_{x\to1^+} \frac{g(x)}{x-1} = \lim_{h\to0} \frac{g(1+h)}{h} = \lim_{h\to0} \frac{\int_1^{1+h} f(t)\,dt}{h}$$

Apply L'Hospital $\beta = \lim_{h\to0} \frac{f(1+h)}{1} = 2$

$\alpha + \beta = 3 + 2 = 5$

**SECTION 4 (Maximum Marks: 12)**

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

**PRAGRAPH "I" (Que. 14 to 15)**

Let S = {1,2,3,4,5,6} and X be the set of all relations R from S to S that satisfy both the following properties :

i. R has exactly 6 elements.

ii. For each (a, b) ∈ R, we have $|a - b| \geq 2$.

Let Y = {R ∈ X : The range of R has exactly one element} and

Z = {R ∈ X : R is a function from S to S}

Let n(A) denote the number of elements in a set A.

**14.** If $n(X) = {}^{m}C_6$, then the value of m is ______ .

**Ans.** **(20)**

**Sol.** Number of elements in S × S = 36

Number of possible ordered pairs which satisfy given condition = 36 – 16 = 20

$n(X) = {}^{20}C_6$

m = 20

**15.** If the value of n(Y) + n(Z) is $k^2$, then |k| is _______.

**Ans.** **(36)**

**Sol.** Maximum number of pre-images of any elements is 4

therefore no six ordered pairs can have same images

n(Y) = 0

All functions have six ordered pairs

number of possible function form S to S

$n(Z) = 4 \times 3 \times 3 \times 3 \times 3 \times 4 = 4^2 \times 3^4$

$n(Y) + n(Z) = 4^2 \times 3^4 = k^2$

k = 36

**PRAGRAPH "II" (Que. 16 to 17)**

Let $f : \left[0, \frac{\pi}{2}\right] \to [0, 1]$ be the function defined by $f(x) = \sin^2 x$ and let $g : \left[0, \frac{\pi}{2}\right] \to [0, \infty)$ be the function defined by $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$.

**16.** The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)\,dx - \int_0^{\frac{\pi}{2}} g(x)\,dx$ is _______.

**Ans.** **(0)**

**Sol.** Let $I_1 = \int_0^{\pi/2} f(x)\,g(x)\,dx = \int_0^{\pi/2} (\sin^2 x)\sqrt{x\left(\frac{\pi}{2} - x\right)}\,dx$ .....(1)

$I_1 = \int_0^{\pi/2} \cos^2 x\sqrt{\left(\frac{\pi}{2} - x\right)x}\,dx$ .....(2)

$2I_1 = \int_0^{\pi/2} (\sin^2 x + \cos^2 x)\sqrt{x\left(\frac{\pi}{2} - x\right)}\,dx$

$2I_1 = \int_0^{\pi/2} \sqrt{x\left(\frac{\pi}{2} - x\right)}\,dx$

$2I_1 - I_2 = 0$

**17.** The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)\,dx$ is ______ .

**Ans.** **(0.25)**

**Sol.** $I_1 = \frac{1}{2}\int_0^{\pi/2} \sqrt{x\left(\frac{\pi}{2}-x\right)}\,dx$

$I_1 = \int_0^{\pi/4} \sqrt{x\left(\frac{\pi}{2}-x\right)}\,dx = \int_0^{\pi/4} \sqrt{\left(\frac{\pi}{4}-x\right)\left(\frac{\pi}{2}-\frac{\pi}{4}+x\right)}\,dx$

$I_1 = \int_0^{\pi/4} \sqrt{\left(\frac{\pi}{4}\right)^2 - x^2}\,dx$

$I_1 = \frac{x}{2}\sqrt{\left(\frac{\pi}{4}\right)^2 - x^2} + \frac{(\pi/4)^2}{2}\sin^{-1}\left(\frac{x}{\pi/4}\right)\Bigg|_0^{\pi/4}$

$I_1 = \left(0+\frac{\pi^2}{32}\times\frac{\pi}{2}\right) - (0+0) = \frac{\pi^3}{64}$

$\frac{16}{\pi^3} I_1 = \frac{1}{4} = 0.25$