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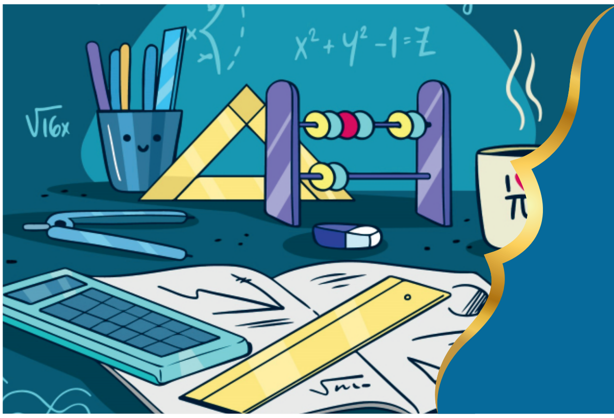
JEE ADVANCED 2023

PAPER - 1



QUESTIONS & SOLUTIONS

4 JUNE, 2023
9:00 AM to 12:00 Noon

Duration : 3 Hours

SUBJECT - MATHEMATICS

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MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –2 marks.

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1. Let $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$. Then which of the following statements is (are) true?

(A) There are infinitely many functions from S to T
(B) There are infinitely many strictly increasing functions from S to T
(C) The number of continuous functions from S to T is at most 120
(D) Every continuous function from S to T is differentiable

Ans. **(ACD)**

Sol. (A) true as domain has infinite elements

(B) False as domain has infinite elements but codomain has only 4 elements. Hence no strictly increasing function is possible

(C) Codomain has finite number of elements

(D) $f'(a) = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

now since $\lim_{h\to 0} f(a+h) = f(a)$ due to continuous

Numerator will be a perfect 0 as output of function is discrete. Hence $f'(a) = 0$ thus differentiable

2. Let T_1 and T_2 be two distinct common tangents to the ellipse E : $\frac{x^2}{6} + \frac{y^2}{3} = 1$ and the parabola P : $y^2 = 12x$. Suppose that the tangent T_1 touches P and E at the points A_1 and A_2, respectively and the tangent T_2 touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is(are) true?

(A) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units

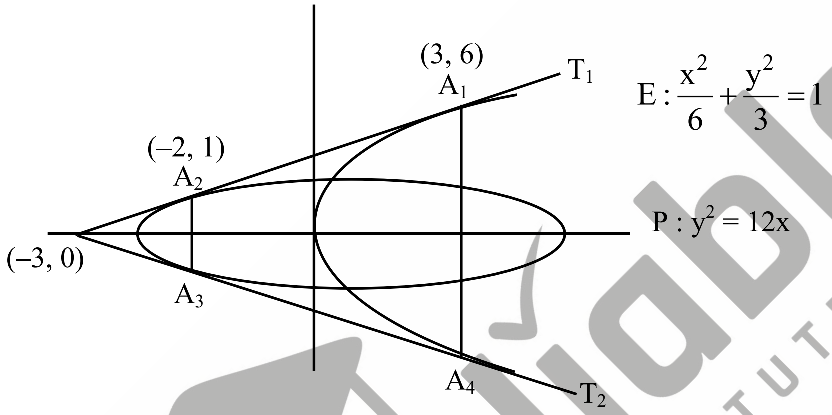
(B) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units

(C) The tangents T_1 and T_2 meet the x -axis at the point (–3,0)

(D) The tangents T_1 and T_2 meet the x -axis at the point (–6,0)

Ans. **(AC)**

Sol.



$y = mx + \sqrt{6m^2 + 3}$

$y = mx + \frac{3}{m}$

$6m^2 + 3 = \frac{9}{m^2} \Rightarrow \quad 2m^4 + m^2 - 3 = 0$

$2m^4 + 3m^2 - 2m^2 - 3 = 0$

$m^2(2m^2 + 3) - (2m^2 + 3) = 0$

$m = \pm 1$

$A_1(3m^2, 6m) = A_1\ (3, 6)$

$\frac{xx_1}{6} + \frac{yy_1}{3} = 1 \Rightarrow xx_1 + 2yy_1 - 6 = 0$ where $A_2 = (x_1, y_1)$

$T_1 : y = x + 3,\ T_2 = y = -x - 3$

T_1 and T_2 intersect at (–3, 0)

$xx_1 + 2yy_1 - 6 = 0$

$x - y + 3 = 0$

$x_1 = -2y_1 = -2$

Area of $A_1A_2A_3A_4 = 2 \times \frac{1}{2}[6 \times 6 - 1 \times 1] = 35$

3. Let $f : [0, 1] \rightarrow [0, 1]$ be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region $S = [0, 1] \times [0, 1]$. Let $G = \{(x, y) \in S : y > f(x)\}$ be called the green region and $R = \{(x, y) \in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h) \in S : x \in [0, 1]\}$ be the horizontal line drawn at a height $h \in [0, 1]$. Then which of the following statements is(are) true?

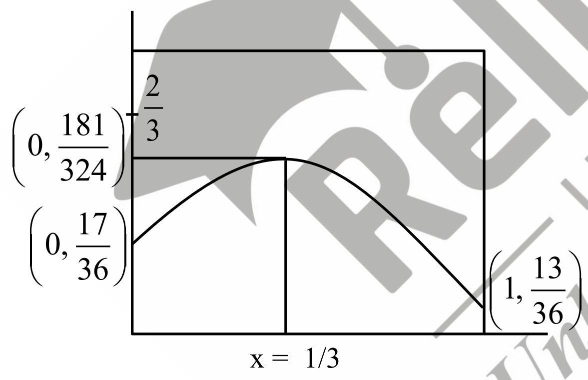
(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

Ans. **(BCD)**



Sol.

$$y = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$$

$$y' = x^2 - 2x + \frac{5}{9}$$

$$9x^2 - 18x + 5 = 0$$

$$\Rightarrow x = \frac{1}{3}, \frac{5}{3}$$

$$y\left(\frac{1}{3}\right) = \frac{181}{324}$$

$$y(1) = \frac{13}{36}$$

Area of green = $1 - \int_0^1 f(x)\,dx = \frac{1}{2}$

(A) $h = \frac{2}{3}$ area of green above L = $\frac{1}{3} > \frac{1}{4}$

Hence (A) wrong

(B) For $h = \frac{1}{4}$ area red below L = $\frac{1}{4}$

Hence (B) correct

(C) For $h = \frac{181}{324}$ area green above L = $\frac{143}{324}$ and red below L = $\frac{1}{2}$ and $h = \frac{13}{36}$

green above = $\frac{1}{2}$ red below = $\frac{13}{36}$

$\therefore$ by IMV the areas are equal some where in between

(D) Check at $h = \frac{181}{324}$ and $h = \frac{13}{36}$ and use IMV

SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

4. Let $f : (0, 1) \to R$ be the function defined as $f(x) = \sqrt{n}$ if $x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$ where $n \in N$. Let $g : (0, 1) \to R$ be a function such that $\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\,dt < g(x) < 2\sqrt{x}$ for all $x \in (0, 1)$. Then $\lim_{x\to 0} f(x)g(x)$

(A) does **NOT** exist (B) is equal to 1 (C) is equal to 2 (D) is equal to 3

Ans. **(C)**

Sol. $x \to 0$

$n \to \infty$

$$\frac{\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\,dt}{\sqrt{x}} < f(x)\,.\,g(x) < \frac{2\sqrt{x}}{\sqrt{x}}$$

$$\left(\frac{\sqrt{\frac{1-x}{x}} - 2x\sqrt{\frac{1-x^2}{x^2}}}{\frac{1}{2\sqrt{x}}}\right) < \lim_{x\to 0} f(x)\,.\,g(x) < 2$$

$$2\sqrt{1-x} - 4\sqrt{x}\ \sqrt{1-x^2} < \lim_{x\to 0} f(x)\,.\,g(x) < 2$$

$$2 < \lim_{x\to 0} f(x)\, g(x) < 2$$

$$\therefore \quad \lim_{x\to 0} f(x)\, g(x) = 2$$

5. Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \in \{0, 1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, d (ℓ_1, ℓ_2) denote the shortest distance between them. Then the maximum value of $d(\ell_1, \ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

Ans. **(A)**

Sol. $\vec{r} = \lambda(1, 1, 1)$

$\vec{r} = (0, 0, 1) + \mu\,(1, 0, -1)$

$$d = \frac{[\vec{a}_1 - \vec{a}_2 \quad \vec{b}_1 \quad \vec{b}_2]}{|\vec{b}_1 \times \vec{b}_2|} = \frac{\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix}}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix}\right|}$$

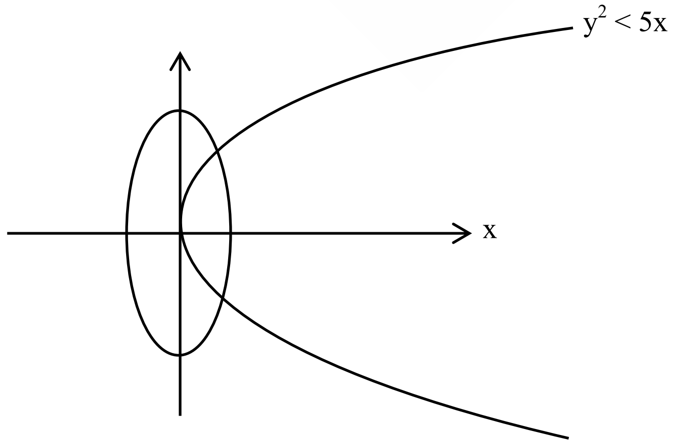
$$d = \frac{1}{\left|-\hat{i} + 2\hat{j} - \hat{k}\right|} = \frac{1}{\sqrt{6}}$$

6. Let $X = \left\{(x, y) \in \mathbb{Z}\times\mathbb{Z} : \frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R from a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$ (C) $\frac{79}{220}$ (D) $\frac{83}{220}$

Ans. **(B)**

Sol.



The points inside are (1, 0), (1, ± 1), (1, ±2), (2, 0), (2, ±1), (2, ±2) & (2, ±3)

Area = integer if height is even

as area = $\frac{1}{2}$. B. H = $\frac{1}{2}$ × 1 × H

H even when ${}^5C_1\,({}^4C_2 + {}^3C_2) + {}^7C_4\,({}^3C_2 + {}^2C_2) = 73$

Total ways = ${}^{12}C_3$

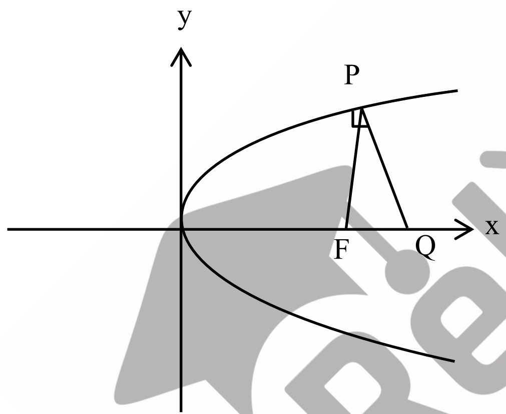
$\therefore$ P = $\frac{73}{220}$

7. Let P be a point on the parabola $y^2 = 4ax$, where $a > 0$. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ , where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a,m) is

(A) (2, 3) (B) (1, 3) (C) (2, 4) (D) (3, 4)

Ans. **(A)**

Sol.



Normal at P : y = mx – 2am – am^3

Q(2a + am^2, 0), P (am^2, –2am)

$$120 = \frac{1}{2}\left|\begin{vmatrix} am^2 & -2am & 1 \\ a & 0 & 1 \\ 2a+am^2 & 0 & 1 \end{vmatrix}\right|$$

240 = |2am (a – 2a – am^2)| = |2am(a + am^2)|

120 = |a^{2}m + a^{2}m^3|

120 = a^{2}m(1 + m^2)

(A) (2, 3) ⇒ 4 × 3 × (1 + 9) = 120 (True)

(B) (1, 3) ⇒ 3 × 10 = 30 (False)

(C) (2, 4) ⇒ 4 × 4 × 17 (False)

(D) (3, 4) ⇒ (False)

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

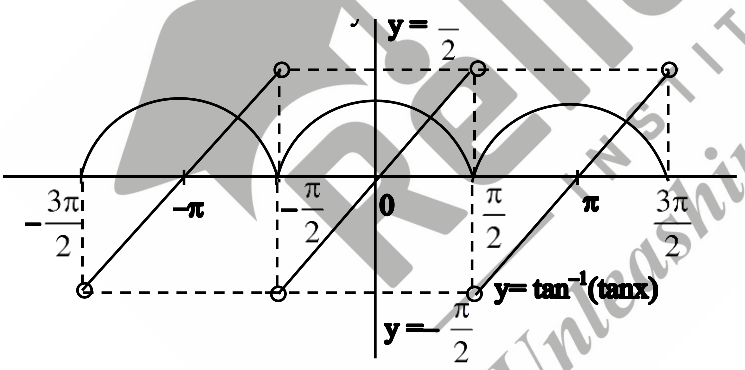
Zero Marks : 0 In all other cases.

8. Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in \mathbb{R}$. Then the number of real solutions of the equation $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

Ans. **(3)**

Sol. $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$

$\Rightarrow \sqrt{2}\,|\cos x| = \sqrt{2}\ \tan^{-1}(\tan x) \Rightarrow |\cos x| = \tan^{-1}(\tan x)$



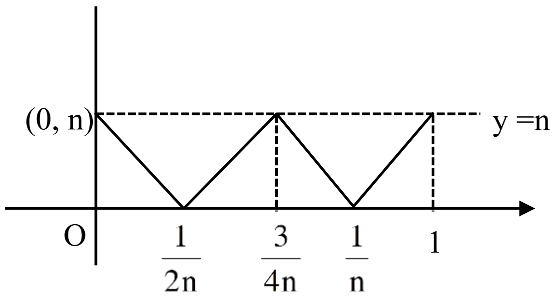
9. Let $n \geq 2$ be a natural number and $f : [0, 1] \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if} \quad 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if} \quad \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if} \quad \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if} \quad \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that area of the region bounded by curves $x = 0$, $x = 1$, $y = 0$ and $y = f(x)$ is 4, then the maximum value of the function f is

Ans. **(8)**

Sol.



$$\text{Area} = \frac{1}{2}\left(\frac{1}{2n}+\frac{1}{n}-\frac{1}{2n}+1-\frac{1}{n}\right)\times n = 4$$

$\Rightarrow n = 8$

now, maximum value of f(x) = n = 8

10. Let $7\overbrace{5\ldots5}^{r}7$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum $S = 77 + 757 + 7557 + \ldots + 7\overbrace{5\ldots5}^{98}7$. If $S = \dfrac{7\overbrace{5\ldots5}^{99}7+m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

Ans. **(1219)**

Sol. $S = 77 + 757 + 7557 + \ldots\ldots + 7\overbrace{55\ldots\ldots5}^{98}7$

$= 70 + 700 \ldots\ldots\ 99 \text{ terms} + 50(1 + 11 + \ldots\ldots\ 98 \text{ terms}) + 7 \times 99$

$= 70\,\dfrac{[10^{99}-1]}{10-1} + \dfrac{50}{9}[(10-1)+(10^2-1)+\ldots\ldots\,(10^{98}-1)] + 7\times 99$

$= 70\dfrac{[10^{99}-1]}{9}+\dfrac{50}{9}\left[10\dfrac{(10^{98}-1)}{10-1}-98\right]+7\times 99$

$= 70\dfrac{[10^{99}]}{9}+\dfrac{50}{9}\left[\dfrac{10^{99}-1}{9}-99\right]+7\times 99-\dfrac{70}{9}$

$= \dfrac{70\times 10^{99}}{9}+\dfrac{50}{9}\left[\dfrac{10^{99}-1}{9}\right]-\dfrac{99\times 50}{9}+7\times 99-\dfrac{70}{9}$

$= \dfrac{70\times 10^{99}}{9}+\dfrac{50}{9}\left[\dfrac{10^{99}-1}{9}\right]+\dfrac{7}{9}+143-\dfrac{77}{9}$

$= \dfrac{755....57+1210}{9}$

m = 1210

n = 9

m + n = 1219

11. Let A = $\left\{\dfrac{1967+1686i\ \sin\theta}{7-3i\cos\theta};\theta\in R\right\}$. If A contains exactly one positive integer n , then the value of n is

Ans. **(281)**

Sol. A = $\dfrac{1967+1686i\sin\theta}{7-3i\cos\theta}=I$

where A is real number

Im (1967 + 1686 i sinθ) (7 + 3i cosθ) = 0

3 × 1967 cosθ + 7 × 1686 sinθ = 0

$\tan\theta = \dfrac{-3\times1967}{7\times1686} = \dfrac{-1}{2}$, $\sin\theta = \dfrac{1}{\sqrt{5}}$ $\cos\theta = \dfrac{-2}{\sqrt{5}}$

or $\sin\theta = \dfrac{-1}{\sqrt{5}}$ $\cos\theta = \dfrac{2}{\sqrt{5}}$

hence A = $\dfrac{1967+1686i\left(\dfrac{1}{\sqrt{5}}\right)}{7+3i\left(\dfrac{2}{\sqrt{5}}\right)}$

$=\dfrac{1967\sqrt{5}+1686i}{7\sqrt{5}+6i}=\dfrac{281\left(7\sqrt{5}+6i\right)}{7\sqrt{5}+6i}$

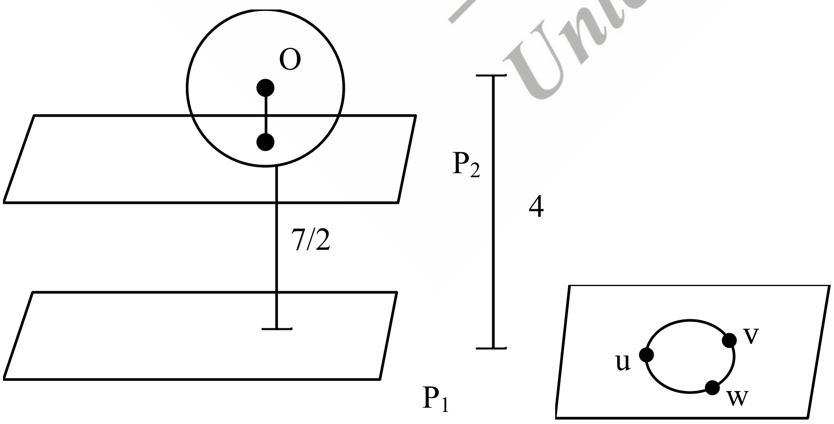
12. Let P be the plane $\sqrt{3}x+2y+3z=16$ and let

$S=\left\{\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}:\alpha^2+\beta^2+\gamma^2=1 \text{ and the distance of } (\alpha,\beta,\gamma) \text{ from the plane P is } \dfrac{7}{2}\right\}$.

Let $\vec{u},\vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u},\vec{v}$ and $\vec{w}$. Then the value of $\dfrac{80}{\sqrt{3}}V$ is .

Ans. **(45)**

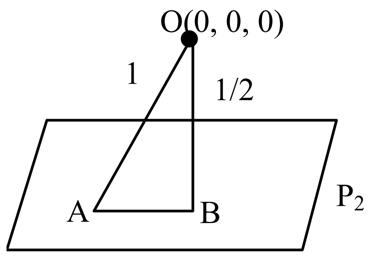
Sol. ⊥ from O to P_1 = 4



sphere with center (0,0,0) & r = 1 is $x^2 + y^2 + z^2 = 1$

$AB^2 = 1 - \dfrac{1}{4}$

$AB = \frac{\sqrt{3}}{2}$



so equilateral Δ is inscribed in this circle of radius $\frac{\sqrt{3}}{2}$

Base Area $= 3\times\frac{1}{2}\times\left(\frac{\sqrt{3}}{2}\right)^2 \sin 120°$

$= \frac{3}{2}\times\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{9\sqrt{3}}{16}$

Volume of pyramid $V = \frac{1}{3} \times h \times$ base area, $V = \frac{1}{6}\times\frac{9\sqrt{3}}{16} = \frac{1}{6}[abc]$

Area of parallelogram $= \frac{9\sqrt{3}}{16} = [abc]$, $\frac{80v}{\sqrt{3}} = 45$

13. Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2+\frac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^7$, then the value of 2b is

Ans. **(3)**

Sol. ${}^4C_r(ax^2)^{4-r}\left(\frac{70}{27bx}\right)^r = T_{r+1}$

${}^4C_r\, a^{4-r}\left(\frac{70}{27b}\right)^r x^{8-3r} = T_{r+1}$

$\therefore$ Coefficient of $x^5 = {}^4C_1\, a^3\left(\frac{70}{27b}\right) = \frac{280a^3}{27b}$

${}^7C_t(ax)^{7-t}\left(\frac{-1}{bx^2}\right)^t = T_{t+1}$

${}^7C_t\, a^{7-t}\left(-\frac{1}{b}\right)^t x^{7-3t} = T_{t+1}$

$\therefore$ Coefficient of $x^{-5} = {}^7C_4\, a^3\left(\frac{1}{b}\right)^4 = {}^7C_4\,\frac{a^3}{b^4}$

$\therefore \frac{280a^3}{27b} = \frac{17}{\lfloor 4 \lfloor 3}\frac{a^3}{b^4} \quad \Rightarrow b^3 = 27/8$

$b = 3/2 \Rightarrow 2b = 3$

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	: 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –1 In all other cases.

14. Let α ,β and γ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List-I** to the correct entries in **List-II.**

	List-I		**List-II**
(P)	If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1)	a unique solution
(Q)	If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2)	no solution
(R)	If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3)	Infinitely many solutions
(S)	If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4)	$x = 11$, $y = -2$ and $z = 0$ as a solution
		(5)	$x = -15$, $y = 4$ and $z = 0$ as a solution

The correct option is :

(A) (P) $\to$ (3) (Q) $\to$ (2) (R) $\to$ (1) (S) $\to$ (4)

(B) (P) $\to$ (3) (Q) $\to$ (2) (R) $\to$ (5) (S) $\to$ (4)

(C) (P) $\to$ (2) (Q) $\to$ (1) (R) $\to$ (4) (S) $\to$ (3)

(D) (P) $\to$ (2) (Q) $\to$ (1) (R) $\to$ (1) (S) $\to$ (3)

Ans. **(A)**

Sol. $D = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 7\alpha - 2\beta - 3$

$D_1 = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix} = -11(2\beta+3) - \alpha(-21-2\gamma)$

$= -22\beta - 33 + 21\alpha + 2\alpha\gamma$

$= 21\alpha - 22\beta + 2\alpha\gamma - 33$

$D_2 = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix} = (11\beta - \alpha\gamma) - 7(\beta - 2\alpha) + 1(\gamma - 22)$

$= 14\alpha + 4\beta + \gamma - \alpha\gamma - 22$

$D_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = -1(2\gamma+21) - 11(-3-4) = -2\gamma - 21 + 77 = -2\gamma + 56.$

at $\beta = \frac{1}{2}(7\alpha - 3)$ & $\gamma = 28$

$D = D_1 = D_2 = D_3 = 0$ Infinite many solution

at $\beta = \frac{1}{2}(7\alpha - 3)$ $\gamma \neq 28$

$D_1 = 0$

$D_3 \neq 0$ no solution

15. Consider the given data with frequency distribution

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in **List-I** to the correct entries in **List-II.**

	List-I		**List-II**
(P)	The mean of the above data is	(1)	2.5
(Q)	The median of the above data is	(2)	5
(R)	The mean deviation about the mean of the above data is	(3)	6
(S)	The mean deviation about the median of the above data is	(4)	2.7
		(5)	2.4

The correct option is:

(A) (P) P $\rightarrow$(3) (Q) $\rightarrow$(2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)

(B) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (4) (S) $\rightarrow$ (1)

(D) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (5) (S) $\rightarrow$ (5)

Ans. **(A)**

Sol.

x_i	3	4	5	8	10	13
f_i	5	4	4	2	2	3
c.f.	5	9	13	15	17	20

Mean = $\dfrac{15+16+33+20+20+16}{20} = 6$

Median = 5

Mean deviation (about mean) = $\dfrac{15+4+15+8+4+8}{20} = \dfrac{54}{20} = 2.7$

Mean deviation (about median) = $\dfrac{10+6+18+10+0+4}{20} = \dfrac{48}{20} = 2.4$

Hence option (A)

16. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2 = (\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.

Match each entry in **List-I** to the correct entries in **List-II**.

List-I		**List-II**	
(P)	The value of $d(H_0)$ is	(1)	$\sqrt{3}$
(Q)	The distance of point (0, 1, 2) from H_0 is	(2)	$\frac{1}{\sqrt{3}}$
(R)	The distance of origin from H_0 is	(3)	0
(S)	The distance of origin from the point of intersection of planes y = z, x = 1 and H_0 is	(4)	$\sqrt{2}$
		(5)	$\frac{1}{\sqrt{2}}$

The correct option is :

(A) (P) $\rightarrow$(2) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (5) (S) $\rightarrow$ (1)

(B) (P) $\rightarrow$ (5) (Q) $\rightarrow$ (4) (R) $\rightarrow$(3) (S) $\rightarrow$ (1)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (3) (S) $\rightarrow$ (2)

(D) (P) $\rightarrow$ (5) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (2)

Ans. **(B)**

Sol. $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$

$\vec{r}_2 = (\hat{j}-\hat{k}) + \mu(\hat{i}+\hat{k})$; passes through (0, 1, –1)

Equation of H_0 :

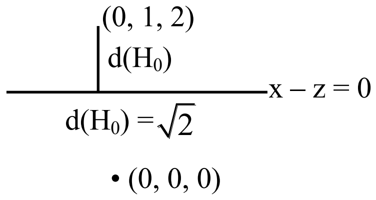
ax + by + cz = 0

$$(a, b, c) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = (1, 0, -1)$$

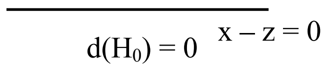
x – z = 0

(P) $d(H_0) = \dfrac{1}{\sqrt{2}}$

(Q)



(R)



(S) Intersection of $x - z = 0$ & $y = z$, $x = 1$ will be $(1, 1, 1)$

$\therefore$ dist. from origin $= \sqrt{3}$

17. Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z . Let the imaginary part of z be non zero.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I		**List-II**	
(P)	$|z|^2$ is equal to	(1)	12
(Q)	$|z-\bar{z}|^2$ is equal to	(2)	4
(R)	$|z|^2 + |z+\bar{z}|^2$ is equal to	(3)	8
(S)	$|z+1|^2$ is equal to	(4)	10
		(5)	7

The correct option is:

(A) (P) → (1) (Q) → (3) (R) → (5) (S) → (4)

(B) (P) → (2) (Q) → (1) (R) → (3) (S) → (5)

(C) (P) → (2) (Q) →(4) (R) → (5) (S) → (1)

(D) (P) → (2) (Q) → (3) (R) → (5) (S) → (4)

Ans. **(B)**

Sol. $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$

$|z|^3 + 2\bar{z}^2 + 4z - 8 = 0$

Subtract $\Rightarrow 2(z^2 - (\bar{z})^2) + 4(\bar{z} - z) = 0$

$2(z - \bar{z})(z + \bar{z} - 2) = 0 \Rightarrow z = \bar{z}$ or $z + \bar{z} = 2$

$y \neq 0 \Rightarrow z + \bar{z} = 2 \Rightarrow x + iy + x - iy = 2 \Rightarrow x = 1$

$\Rightarrow \quad |1 + iy|^3 + 2(1 + iy)^2 + 4(1 - iy) - 8 = 0$

$\Rightarrow \quad (1 + y^2)^{3/2} + 2(1 - y^2 + 2iy) - 4 - 4iy = 0$

$(1 + y^2)^{3/2} + 2 - 2y^2 - 4 = 0,\ i(4y - 4y) = 0$

$(1 + y^2)^{3/2} - 2(1 + y^2) = 0 \Rightarrow \sqrt{1+y^2} = 2 \Rightarrow y^2 = 3$

(P) $|z|^2 = x^2 + y^2 = 4$

(Q) $|z - \bar{z}|^2 = 4y^2 = 12$

(R) $|z|^2 + |z + \bar{z}|^2 = x^2 + y^2 + 4x^2 = 5x^2 + y^2 = 8$

(S) $|z + 1|^2 = (x + 1)^2 + y^2 = 4 + 3 = 7$

Option (B) is correct