

JEE(Advanced) 2022

Memory Based Paper

PAPER-1 & 2

Questions & Solutions

Date : 28 August, 2022

SUBJECT : PHYSICS

SECTION-I

1. In a particular system a of units, a physical quantity can be expressed in terms of the electric charge e , electron mass m_e , Planck's constant h , and coulombs constant $K = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is permittivity of vacuum, In terms of these physical constants, the dimension of the magnetic field is $[B] \propto [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$. the value of $\alpha + \beta + \gamma + \delta$ is

Ans. 4

Sol. $[B] = [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$

Dimension formula for magnetic field.

$$F = qvB$$

$$\frac{[MLT^{-2}]}{[C][LT^{-1}]} = B = [MT^{-1}C^{-1}]$$

Dimension of electron charge $[e]^\alpha = [c]^\alpha$

Dimension of mass of electron $[m_e] = [M]^\beta$

Dimension of Planck's constant h [unit is Js]

$$[h] = [ML^2T^{-2}][T] = [ML^2T^{-1}]^\gamma$$

Dimension K ' coulombs constant

$$F = \frac{kQ^2}{r^2} \quad K = \frac{[MLT^{-2}][L^2]}{[C]^2}$$

$$K = [ML^3T^{-2}C^{-2}]^\delta$$

For dimensional constancy.

$$[MT^{-2}C^{-1}] = [C]^\alpha [M]^\beta [ML^2T^{-1}]^\gamma [ML^3T^{-2}C^{-2}]^\delta$$

$$[M]^1 [L]^0 [T]^{-1} [C]^{-1} = [C]^{\alpha-2\delta} [M]^{\beta+\gamma+\delta} [L]^{2\gamma+3\delta} [T]^{-\gamma-2\delta}$$

$$2\gamma + 3\delta = 0 \quad \dots(1)$$

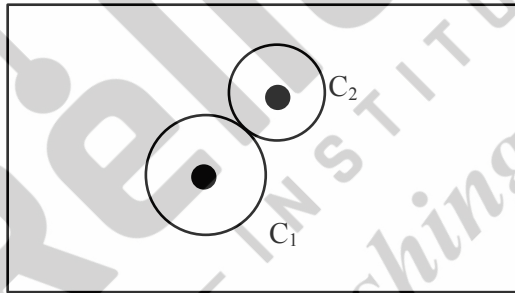
$$-\gamma - 2\delta = -1 \quad \dots(2)$$

$$\beta + \gamma + \delta = 0 \quad \dots(3)$$

$$\alpha - 2\delta = -1 \quad \dots(4)$$

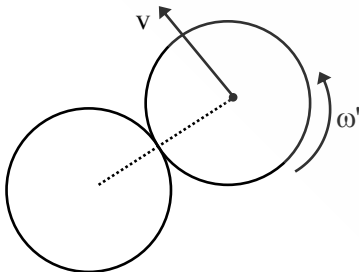
$$\alpha + \beta + \gamma + \delta = 4$$

2. Disc C_1 of radius R is fixed on horizontal table by its flat surface and another disc C_2 of radius R does pure rolling on C_1 . Angular momentum of C_2 wrt to centre of C_1 is $nm\omega^2R$. Find $n = ?$



Ans. 5

Sol. Assuming angular velocity about centre as ω'



$$\frac{v}{2R} = \omega$$

$$\text{and } \omega' = v/R$$

$$\text{so } \omega' = 2\omega$$

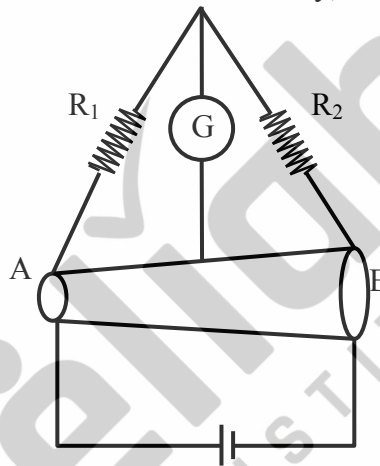
hence, $\vec{L} = \vec{L}_{CM} + m(\vec{r} \times \vec{v})$

$$|L| = \frac{mR^2}{2} \omega' + mR(\omega'R)$$

$$= \frac{mR^2}{2} \times 2\omega + M \times 2R(2\omega)R = 5 MR^2 \omega$$

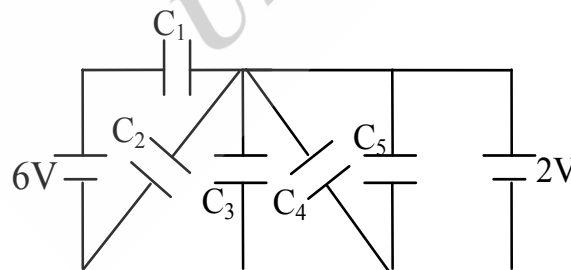
n = 5

3. Two resistances $R_1 = x\Omega$ and $R_2 = 1\Omega$ are connected to a wire AB of uniform resistivity as shown in the figure. The radius of the wire varies linearly along its axis from 0.2 mm at A to 1 mm at B. A galvanometer (G) connected to the center of the wire, 50 cm from each end along its axis shows zero deflection when A and B are connected to a battery, the value of X is _____

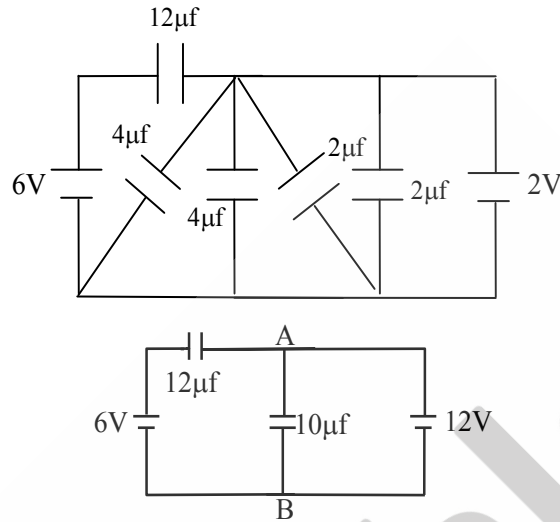


Ans. 5

4. In the following circuit $C_1 = 12\mu f$, $C_2 = C_3 = 4\mu f$ and $C_4 = C_5 = 2\mu f$ the charge scored in C_3 is _____ μC .



Sol.

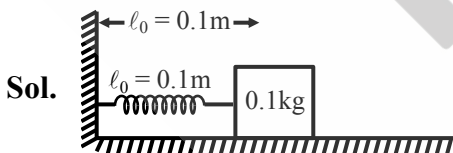


$$V_A - V_B = 2V$$

$$\text{Charge on } C_3 = 2 \times 4 = 8\mu\text{C}$$

5. On a frictionless horizontal plane, a bob of mass $m = 0.1 \text{ kg}$ is attached to a spring with natural length $\ell_0 = 0.1 \text{ m}$. The spring constant is $k_1 = 0.009 \text{ Nm}^{-1}$ when the length of the spring $\ell > \ell_0$ and is $k_2 = 0.016 \text{ Nm}^{-1}$ when $\ell < \ell_0$. Initially the bob is released from $\ell = 0.15 \text{ m}$. Assume that Hooke's law remain valid throughout the motion. If the time period of the full oscillation is $T = (n\pi)\text{s}$, then the integer closest to n is

Ans. 6



if $\ell > \ell_0$

$$K_1 = 0.009 \text{ N/m}$$

$$T_1 = \pi \sqrt{\frac{m}{K_1}}$$

if $\ell < \ell_0$

$$k_2 = 0.016 \text{ N/m}$$

$$T_2 = \pi \sqrt{\frac{m}{K_2}}$$

$$\text{Total time period } T = T_1 + T_2$$

$$T = \pi\sqrt{m} \left[\frac{1}{\sqrt{0.009}} + \frac{1}{\sqrt{0.16}} \right]$$

$$T = \frac{35\pi}{6} \text{ sec} = n\pi \text{ sec}$$

$$n = \frac{35}{6} \approx 6$$

6. A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j})$ kg ms^{-2} with $k = 1 \text{ kgs}^{-2}$. At time $t = 0$, the particle position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j} \right)$ m and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k} \right) \text{ms}^{-1}$. Let v_x and v_y denote the x and the y components of the particles velocity respectively. Ignore gravity. When $z = 0.5$ m, the value of $(xv_y - yv_x)$ is m^2s^{-1} .

Ans. 3

Sol. in $A_2 = 0$ $V_2 = \frac{2}{\pi} = \text{const}^n$

$$S_2 = V_2 t$$

$$0.5 = \frac{2}{\pi} t \Rightarrow t = \frac{\pi}{4} \text{ sec}$$

Now $\vec{F} = -x\hat{i} - y\hat{j} = m(a_x\hat{i} + a_y\hat{j})$

In x-dirⁿ

$$a_x = -x \quad [\text{Motion is SHM}]$$

$$V_x \times \frac{dV_x}{dx} = -x \int_{-\sqrt{2}}^{V_x} V_x dV_x = - \int_{\frac{1}{\sqrt{2}}}^x x dx$$

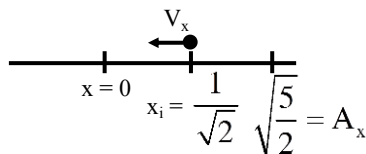
$$V_x^2 - 2 = -x^2 + \frac{1}{2}$$

$$V_x = \sqrt{\frac{5}{2} - x^2}$$

$$\omega = 1 \text{ and } A_x = \sqrt{\frac{5}{2}}$$

at $t = 0$

eqⁿ of SHM



$$x = \sqrt{\frac{5}{2}} \sin(t + \phi)$$

$$\text{at } t = 0 \quad x = \frac{1}{\sqrt{2}} \text{ then } \sin\phi = \frac{1}{\sqrt{5}}$$

$$\cos\phi = -\frac{2}{\sqrt{5}}$$

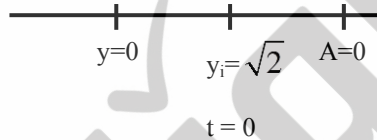
$$\text{at } t = \frac{\pi}{4} \quad x = \sqrt{\frac{5}{2}} \sin\left[\frac{\pi}{4} + \phi\right]$$

$$x = -\frac{1}{2} \text{ and } V_x = -\frac{3}{2} \text{ m/s}$$

$$\text{In } y\text{-dir}^n \quad -y = V_y \frac{dV_y}{dy}$$

$$\int_{\sqrt{2}}^{V_y} V_y dV_y = -\int_{\sqrt{2}}^y g dy$$

$$V_y = \sqrt{4 - y^2}$$



$$A_y = 2 \text{ \& } \omega = 1$$

$$y = 2 \sin \omega = 1$$

$$\text{at } t = 0 \quad y = \sqrt{2} \quad \phi = \frac{\pi}{4}$$

$$y = 2 \sin\left(t + \frac{\pi}{4}\right)$$

$$\text{at } t = \frac{\pi}{4} \quad y = 2 \quad V_y = \sqrt{y^2 - y^2} = 0$$

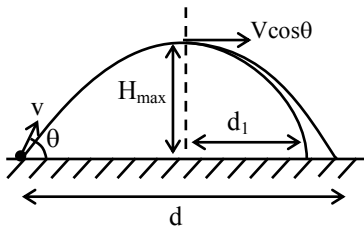
$$[xV_y - yV_x] = 3 \text{ Ans.}$$

7. A projectile is thrown with velocity V at inclination of θ from horizontal and its range is d in presence of gravity g . Same projectile is now thrown at same angle θ and same velocity V but after covering half of the range ($d/2$), g' is acting. And now the total range of particle becomes d' . If $g' =$

$$\frac{g}{(0.81)} \text{ then } d' = nd, \text{ find } n?$$

Ans. 0.95

Sol. $d = \frac{2v_x v_y}{g}$



$$H_{\max} = \frac{v_y^2}{2g}, \quad d_1 = v_x \sqrt{\frac{2H_{\max}}{g}}$$

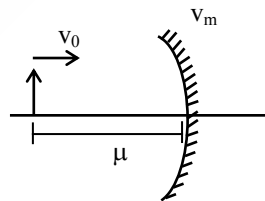
$$d_1 = v_x \sqrt{\frac{2v_y^2 \times 0.81}{2g}}$$

$$d_1 = \frac{v_y \cdot v_x}{g} \times 0.9 = \frac{0.9d}{2}$$

$$d_{\text{Total}} = \frac{d}{2} + \frac{9d}{20} = \frac{19d}{20}$$

$$n = \frac{19}{20} = \boxed{0.95}$$

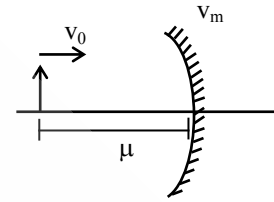
8. An object and a concave mirror of focal length $f = 10$ cm both move along the principal axis of the mirror with constant speed. The object move with speed $v_o = 15 \text{ cm s}^{-1}$ towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by μ when $\mu = 30$ cm the speed of the mirror v_m is such that the image is instantaneously at rest with respect to the laboratory frame and the object forms a real image. The magnitude of v_m is _____ cm s^{-1}



Ans. 3

Sol. $u = -30 \text{ cm}$ $f = -10 \text{ cm}$

$$m = \frac{f}{f - u}$$



$$m = \frac{-10}{-10 - (-30)} = \frac{-10}{30 - 10}$$

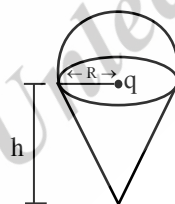
$$m = -\frac{1}{2}$$

$$\vec{V}_1 = -m^2 \vec{V}_0 + (m^2 + 1) \vec{V}_m$$

$$0 = \left(-\frac{1}{2}\right)^2 \times 15 + \left(\left(-\frac{1}{2}\right)^2 + 1\right) \times V_m$$

$$\frac{5}{4} V_m = \frac{15}{4} \Rightarrow V_m = 3 \text{ cm/s}$$

9. A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R and a hemisphere of radius R as shown in the figure the electric flux through the conical surface is $\frac{nq}{6\epsilon_0}$ (in SI units) the value of n is _____



Ans. 3

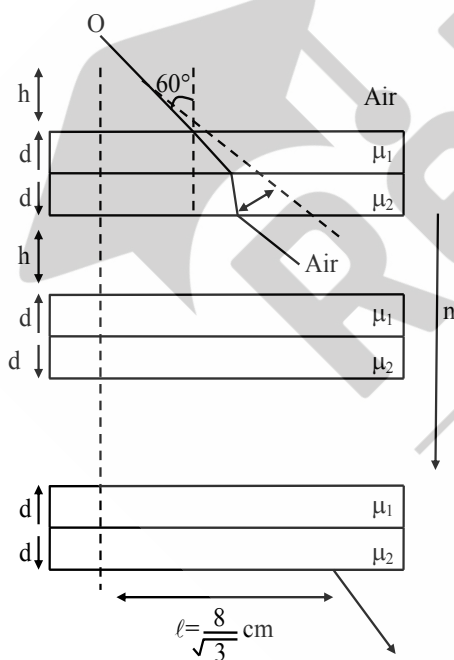
Sol. $\frac{q}{2\epsilon_0} = \frac{nq}{6\epsilon_0}$

$$n = 3$$

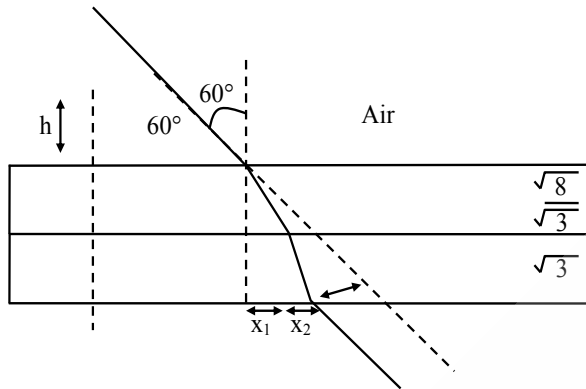
10. Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3}$ cm and the second & third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2}$ cm, and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$ respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $Q = 60^\circ$ to the normal, for a specific value of n , the ray of light emerges from the bottom of the configuration at a stance $\ell = \frac{8}{\sqrt{3}}$ cm as shown in the figure, the value of n is

Ans. 4

Sol.



$$\Delta = d \frac{\sin(i - r_1)}{\cos r_1} + d \frac{\sin(i - r_2)}{\cos r_2}$$



$$\sin 60 = \frac{\sqrt{3}}{\sqrt{2}} \sin r_1$$

$$r_1 = 45^\circ$$

$$\& \sin 60 = \sqrt{3} \sin r_2$$

$$\frac{\sqrt{3}}{2} = \sqrt{3} \sin r_2$$

$$r_2 = 30^\circ$$

Now,

Due to 1st slab (arrangement)

$$\Delta : h \tan 60^\circ + d \tan 45^\circ + d \tan 30^\circ$$

$$= \frac{1}{3} \times \sqrt{3} + \left(\frac{\sqrt{3}-1}{2} \right) \left(1 + \frac{1}{\sqrt{3}} \right)$$

$$= \frac{1}{\sqrt{3}} + \left(\frac{(\sqrt{3}-1)(\sqrt{3}+1)}{2\sqrt{3}} \right)$$

$$= \sqrt{\frac{1}{3}} + \frac{2}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

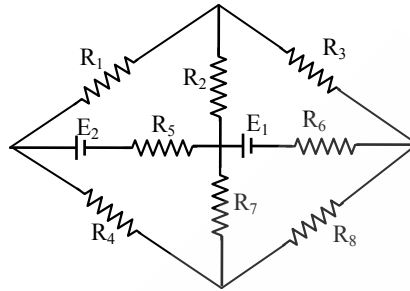
So,

$$l = n\Delta$$

$$\text{So, } = \sqrt{\frac{8}{3}} = n \left(\frac{2}{\sqrt{3}} \right)$$

$$\therefore n = 4$$

11. All resistance are 1Ω . Find current in each resistance :-



Ans.

12. Lc circuit with $L = 0.1 \text{ H}$ and $C = 10^{-3} \text{ F}$, Area of the circuit = 1 m^2

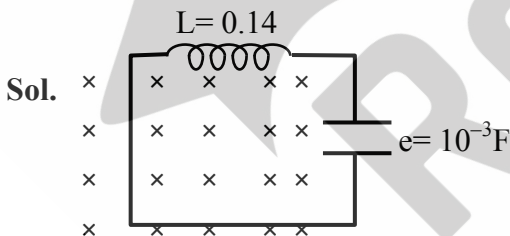
magnetic field is charging as $B = B_0 + Bt$

where $B = 0.4 \text{ T/sec}$

Find max current through circuit

At $t = 0, I = 0$

Ans. 0.04



$$\phi = BA = (B_0 + Bt) \quad (1)$$

$$t = \frac{dB}{dt} = 0.4$$

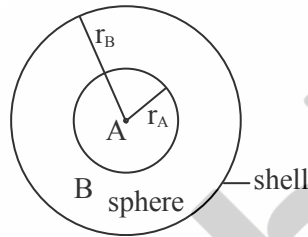
$$\# \text{ For } I = I_{\max} \Rightarrow \frac{dI}{dt} = 0, \frac{Ldi}{dt} = 0$$

$$W_b = BV_C + BV_L$$

$$\# (C) E = \frac{1}{2} Li^2 + \frac{1}{2} CE^2 \Rightarrow I = \sqrt{\frac{C}{L}} (E) = 0.04 \text{ A}$$

13. In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatics charge density varies with radial distance r from the centre as $\rho_A = kr$. Where k is positive. In the spherical shell B of outer radius r_B . The electrostatics charge density varies as

$$\rho_B = \frac{2k}{r}$$



- (A) $r_B = \frac{3}{2}$, V just outside B is $\frac{k}{\epsilon_0}$
- (B) If $r_B = \sqrt{3/2}$ then Electric field is zero every where outside B
- (C) If $r_B = 2$, then total charge of configuration is $15\pi k$
- (D) If $r_B = \frac{5}{2}$ then the magnitude of electric field just outside B is $\frac{13\pi k}{\epsilon_0}$

Sol. $Q = \int_0^1 kx 4\pi x^2 dx + Q = \int_0^{r_B} \frac{2k}{x} 4\pi x^2 dx$

$$4\pi k \left[\frac{1}{4} \right] + \frac{8\pi k}{2} [r_B^2 - 1]$$

$$Q = 4\pi k \left[\frac{1}{4} + r_B^2 - 4 \right]$$

(A) $r_B = \frac{3}{2}, Q = 4\pi k \left[\frac{1}{4} + \frac{9}{4} - 4 \right] = 4\pi k \left[\frac{6}{4} \right] = 6\pi k \quad V = \frac{k}{3/2} (6\pi k) = \frac{k}{3} \left(6\pi \cdot \frac{1}{4} \right) = \frac{k}{60}$

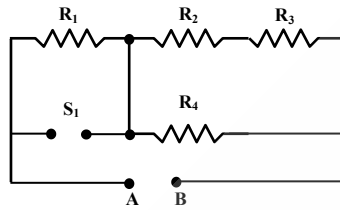
(B) $r_B = \sqrt{3/2}, Q = 4\pi k \left[\frac{1}{4} + \frac{3}{2} - 4 \right] = 4\pi k \left[\frac{1+6-4}{4} \right] = 3\pi k \quad E \neq 0$

(C) $Q = 4\pi k \left[\frac{1}{4} + 4 - 4 \right] = 4\pi k \left[\frac{1+12}{4} \right] = 13\pi k$

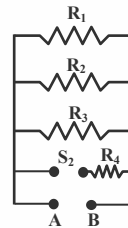
(D) $Q = 4\pi k \left[\frac{1}{4} + \frac{25}{4} - 4 \right] = 4\pi k \left[\frac{26-4}{4} \right] = 22\pi k \quad E = \frac{KQ}{r_B^2} = \frac{KQ}{25/4} = \frac{1}{4/100} \times 4 = \frac{22K}{2500}$

14. $R_1 = 1 \Omega$, $R_2 = 2 \Omega$, $R_3 = 3 \Omega$, $R_4 = \frac{1}{2} \Omega$

Circuit-1



Circuit-2

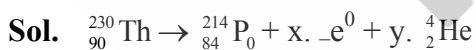


Let P & Q be the power dissipated when (s_1, s_2) open & (s_1, s_2) close respectively.

- (A) If $V_{AB} = 6$ Volt then $P_1 < P_2$
- (B) If $I = 2$ Ampere then $Q_2 > Q_1$
- (C) If $I = 2$ Ampere then $Q_1 > Q_2$
- (D) If $V_{AB} = 6$ Volt then $P_1 < Q_1$

15. In a radioactive decay chain reaction ${}_{90}^{230}\text{Th}$ nucleus decay into ${}_{84}^{214}\text{Pb}$ nucleus. The ratio of number of α to no. of β^- particles emitted in this process is?

Ans. 2



$$90 = 84 - x + 2y \quad \dots\dots(i)$$

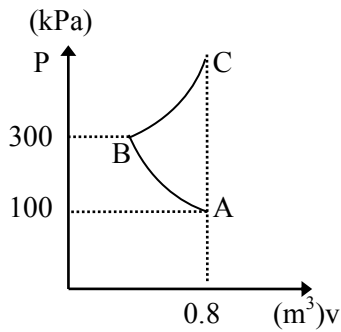
$$230 = 214 + 4x \quad \dots\dots(ii)$$

$$xy = 16, \quad \boxed{y = 4}$$

$$\boxed{x = 2}$$

$$\frac{y}{x} = 2$$

16.



AB → adiabatic

BC → isothermal

$$r = \frac{5}{3}$$

options

$$W_{AB} = ?$$

$$W_{CA} = ?$$

$$W_{ABC} = ?$$

Sol. (i) $W_{AB} = \frac{P_i V_i - P_f V_f}{r - 1} = \frac{100 \times 0.8 \times 10^3 - 300 \times 0.4 \times 10^3}{\frac{2}{3} - 1} = \frac{10^5 (0.4) \times 3}{2} = 6 \times 10^4$

$$\Rightarrow W_{AB} = -60 \text{ kJ}$$

$$100(0.8)^{5/3} = 300 V^{5/3}$$

$$\left(\frac{1}{3}\right)^{3/5} (0.8) = V$$

$$V = 0.4$$

$$W_{CA} = 0$$

$$(iii) w = nRT \ln \frac{0.8}{0.4}$$

$$nRT \ln 2$$

$$= 300 \times 10^3 \times 0.4 \ln 2$$

$$= 120 \ln 2$$

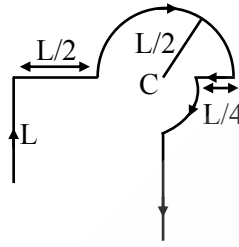
$$W_{ABC}$$

$$= 120 \ln 2 - 60$$

$$= 120 \times 0.693 - 60$$

$$= 23.16 \text{ kJ}$$

17. Find magnetic field at center 'C'.



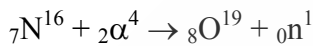
- (A) $\frac{\mu_0 I}{4} \left[\frac{1}{4\sqrt{2}\pi} + 1 \right]$ (B) $\frac{\mu_0 I}{4} \left[\frac{1}{4\sqrt{2}\pi} + \frac{3}{2} \right]$ (C) $\frac{\mu_0 I}{4} \left[\frac{1}{4\sqrt{2}\pi} + 2 \right]$ (D) $\frac{\mu_0 I}{4} \left[\frac{1}{4\sqrt{2}\pi} + \frac{1}{2} \right]$

Ans. (A)

Sol. $\vec{B} = \frac{\mu_0 I}{4\pi L} \left(\frac{1}{\sqrt{2}} \right) + \frac{\mu_0 I}{4 \times L/2} + \frac{\mu_0 I}{8 \times L/4}$

$$\vec{B} = \frac{\mu_0 I}{L} \left(\frac{1}{4\pi\sqrt{2}} + 1 \right)$$

18. α -particle bombards N at rest and following reaction occurs.



Find minimum kinetic energy of α -particle of reaction to occur.

$$M_N = 16.006 \text{ amu}, M_\alpha = 4.003 \text{ amu}, M_O = 19.003 \text{ amu}$$

$$M_n = 1.008 \text{ amu and } 1 \text{ amu} = 930 \text{ Mev}/c^2$$

Ans. 2.35

Sol. Q value = $[M_O + M_n - (M_N + M_\alpha)] \times 930 \text{ Mev}$

$$= [(19.003 + 1.008) - (16.006 + 4.003)] \times 930 \text{ Mev}$$

$$= 1.86 \text{ Mev}$$

By momentum and energy conservation

$$\text{Minimum KE of } \alpha = \left[1 + \frac{m}{M} \right] Q$$

$$\text{KE}_{\min} = \left[1 + \frac{4}{16} \right] \times 1.86$$

$$= 2.35 \text{ Mev}$$