MATHEMATICS

1. If $\frac{4z^2+3z+2}{4z^2-3z+2}$ is a real number and $z \neq \bar{z}$ then find the value of $|z|^2$.

Sol. $w = \frac{4z^2+3z+2}{4z^2-3z+2} = 1 + \frac{6}{4z+\frac{2}{z}-3}$

$$4z+\frac{2}{z} = 4\bar{z}+\frac{2}{\bar{z}} \Rightarrow 4\left(z-\bar{z}\right)+2\left(\frac{1}{z}-\frac{1}{\bar{z}}\right)=0 \Rightarrow 4-\frac{2}{|z|^2}=0 \Rightarrow |z|^2=\frac{1}{2}$$

2. If $f(x) = \sin\left(\frac{\pi x}{12}\right)$ and $g(x) = \frac{2\ell n\left(\sqrt{x}-\sqrt{\alpha}\right)}{\ell n\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}$ then evaluate $\lim\limits_{x\to\alpha^+} f(g(x))$

(A) 2 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) 3

Ans. (C)

Sol. $\lim\limits_{x\to\alpha^+} \frac{2\log\left(\sqrt{x}-\sqrt{\alpha}\right)}{\log\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}$ Apply LH Rule

$$= \frac{\frac{2}{\left(\sqrt{x}-\sqrt{\alpha}\right)}\frac{1}{2\sqrt{x}}}{\frac{1}{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}\times e^{\sqrt{x}}\times\frac{1}{2\sqrt{x}}} = \lim_{x\to\alpha^+} 2\left(\frac{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}{\sqrt{x}-\sqrt{\alpha}}\right)\frac{1}{e^{\sqrt{x}}} = \lim_{x\to\alpha^+} 2\frac{\left(e^{\sqrt{x}-\sqrt{\alpha}}-1\right)}{\sqrt{x}-\sqrt{\alpha}}\frac{e^{\sqrt{\alpha}}}{e^{\sqrt{x}}} = 2$$

$$\Rightarrow \lim_{x\to\alpha^+} f(g(x)) = f(2) = \sin\frac{\pi}{6} = \frac{1}{2}$$

3. Find the value of : (In only principal arguments)

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{\pi^2+2}} + \frac{1}{4}\sin^{-1}\left(\frac{2\sqrt{2}\pi}{\pi^2+2}\right) + \tan^{-1}\left(\frac{\sqrt{2}}{\pi}\right)$$

Ans. $\frac{3\pi}{4}$

Sol. $\frac{3}{2}\tan^{-1}\frac{\pi}{\sqrt{2}} + \frac{1}{4}\sin^{-1}\left(\frac{2\frac{\pi}{\sqrt{2}}}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}\right) + \cot^{-1}\left(\frac{\pi}{\sqrt{2}}\right) = \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\frac{\pi}{\sqrt{2}} + \frac{1}{4}\sin^{-1}\left(\frac{2\frac{\pi}{\sqrt{2}}}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}\right)$

$$\tan^{-1}\frac{\pi}{\sqrt{2}} = \theta \Rightarrow \tan\theta = \frac{\pi}{\sqrt{2}} = \frac{\pi}{2} + \frac{1}{2}\theta + \frac{1}{4}\sin^{-1}\sin 2\theta = \frac{\pi}{2} + \frac{1}{2}\theta + \frac{1}{4}(\pi - 2\theta) = \frac{3\pi}{4}$$

4. Let $\bar{z} - z^2 = i\left(\bar{z}+z^2\right)$; then number of distinct solution is -

Ans. **(4)**

Sol. $(1-i)\,\bar{z}=(1+i)z^2$ ... (i)

$\sqrt{2}\,|z|=\sqrt{2}\,|z|^2$

$|z|=0$ $\qquad$ $|z|=1$

$\Rightarrow z=0$ $\qquad$ $\Rightarrow z=e^{i\theta}$

From equation (i)

$\sqrt{2}\,e^{\frac{-i\pi}{4}}e^{-i\theta}=\sqrt{2}\,e^{i\left(\frac{\pi}{4}+2\theta\right)}$

$\Rightarrow\ e^{-3i\theta}=i=e^{i\left(2k\pi+\frac{\pi}{2}\right)}$

$\Rightarrow \theta=\dfrac{-1}{3}\left(2k\pi+\dfrac{\pi}{2}\right)$

$\therefore$ k = 0, 1, 2

$\therefore$ No of distinct solutions = 4

5. Find the number of integers which can be formed using the digits 0,2,3,4,6 & 7 which lie in the range [2022 , 4482]

Ans. **(569)**

Sol. [2][0][ ][ ] = 36 – [2][0][0][ ] + [2][0][2][0] = 29

6×6

[3][ ][ ][ ] $6\times6\times6=216$

[2][ ][ ][ ] =180

$5\times6\times6$

[4][ ][ ][ ] $=6\times6\times3=108$

[4][4][ ][ ] $6\times6=36$

Total = 216 + 36 + 180 + 108 + 36 – 7 = 569

6. If $x^{16(\log_5 x)^3-68\log_5 x}=5^{-16}$, then product of roots of this equation will be

Ans. (1)

Sol. Taking logarithm to the base 5

$\left\{16(\log_5 x)^3-68(\log_5 x)\right\}\ (\log_5 x)=-16$

let $\log_5 x=t$

$16t^4-68t^2+16=0$

$4t^4-17t^2+4=0$

$4t^4-16t^2-t^2+4=0$

$(4t^2-1)(t^2-4)=0$

$t=\dfrac{1}{2},-\dfrac{1}{2},2,-2$

$\log_5(x_1x_2x_3x_4)=0$

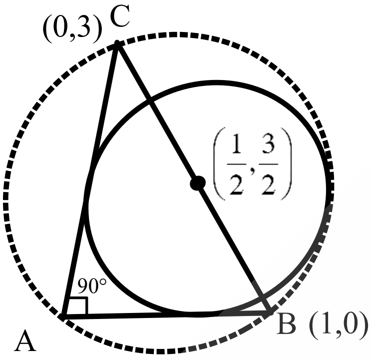
$x_1x_2x_3x_4=1$

7. Let ABC be a right angle triangle where $\angle A = 90°$, side AB = 1 and AC = 3. If a circle formed which touches the sides AB and AC and also touches the circumcircle of ΔABC internally then its radius is

(A) $4-\sqrt{10}$ (B) $4+\sqrt{10}$ (C) $5-\sqrt{10}$ (D) $5+\sqrt{10}$

Ans. (A)

Sol.



Let centre be (α,α)

$C_1C_2 = |r_1 - r_2|$

$$\sqrt{\left(\alpha-\frac{1}{2}\right)^2+\left(\alpha-\frac{3}{2}\right)^2}=\left|\alpha-\frac{\sqrt{10}}{2}\right|$$

$$2\alpha^2+\frac{1}{4}+\frac{9}{4}-\alpha-3\alpha=\alpha^2+\frac{10}{4}-\sqrt{10}\alpha$$

$$\alpha^2-4\alpha+\sqrt{10}\alpha=0$$

$\alpha = 0$ or $\alpha = 4-\sqrt{10}$

Hence $r = 4-\sqrt{10}$

8. Let $M = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$, then matrix M^{2022} is equal to

(A) $\begin{bmatrix} 3034 & 3033 \\ -3033 & -3032 \end{bmatrix}$ (B) $\begin{bmatrix} 3034 & 3033 \\ 3033 & -3032 \end{bmatrix}$

(C) $\begin{bmatrix} 2034 & 2033 \\ -2033 & -2032 \end{bmatrix}$ (D) $\begin{bmatrix} 2034 & 2033 \\ 2033 & -2032 \end{bmatrix}$

Ans. **(A)**

Sol. $M = \begin{bmatrix} 1+\frac{3}{2} & \frac{3}{2} \\ -\frac{3}{2} & 1-\frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{3}{2} \end{bmatrix} = I + \frac{3}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$M = I + \frac{3}{2}A$

$$A^2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A^3 = A^4 = ... = O$

$$M^{2022} = \left(I + \frac{3}{2}A\right)^{2022} = I + 2022\ \frac{3}{2}A + O + O\ .... = I + 1011 \times 3A = I + 3033A$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3033 & 3033 \\ -3033 & -3033 \end{bmatrix} = \begin{bmatrix} 3034 & 3033 \\ -3033 & -3032 \end{bmatrix}$$

9. Consider an AP $< a_n >$ with initial term 7 and common difference 8. Define T_n such that $T_{n+1} - T_n = a_n$. Then (given $T_1 = 3$)

(A) $T_{30} = 9986$ (B) $T_{20} = 1504$

(C) $\sum_{n=1}^{30} T_n = 35615$ (D) $\sum_{n=1}^{20} T_n = 9460$

Ans. (B,D)

Sol. $\sum_{r=1}^{n}(T_{r+1} - T_r) = \sum_{r=1}^{n} a_r$

$T_{n+1} - T_1 = \frac{n}{2}[14 + (n-1)8]$

$T_{n+1} - T_1 = n(3 + 4n)$

$T_{n+1} = 4n^2 + 3n + 3$

$\Rightarrow T_{20} = 4(19)^2 + 3(19) + 3 = 1504$

Now, $T_n = 4(n-1)^2 + 3(n-1) + 3 = 4n^2 - 5n + 4$

$$\sum_{n=1}^{20} T_n = \frac{4 \times 20 \times 21 \times 41}{6} - \frac{5 \times 20 \times 21}{2} + 4 \times 20$$

$= 1180 - 2100 + 80$

$= 9460$

10. Consider two AP's having first terms ℓ_1 & w_1 respectively & common difference d_1 & d_2 respectively. If $d_1d_2 = 10$ & we define A_i to be area of a rectangle whose length is ℓ_i & width w_i. Also $A_{51} - A_{50} = 1000$ then value of $A_{100} - A_{90}$ is -

Ans. (18900)

Sol. $\ell_{51}w_{51} - \ell_{50}w_{50} = 1000$

$(\ell_1 + 50d_1)(w_1 + 50d_2) - (\ell_1 + 49d_1)(w_1 + 49d_2) = 1000$

$\ell_1 d_2 + w_1 d_1 + (50^2 - 49^2)d_1 d_2 = 1000$

$\ell_1 d_2 + w_1 d_1 + 99 d_1 d_2 = 1000$ ...(i)

Consider $A_{100} - A_{90} = (\ell_1 + 99d_1)(w_1 + 99d_2) - (\ell_1 + 89d_1)(w_1 + 89d_2)$

$= 10\ell_1 d_2 + 10d_1w_1 + (99^2 - 89^2)\, d_1d_2$

$= 10\ell_1 d_2 + 10d_1w_1 + 10 \times 188\, d_1d_2$

$= 10(\ell_1 d_2 + d_1w_1 + 188\, d_1d_2)$

$= 10(1000 + 89d_1d_2)$

$= 18900$

11. $\int_1^e \frac{(\ell nx)^{1/2}}{x\left(a-(\ell nx)^{3/2}\right)^2}dx = 1$,

Then

(A) A possible value of 'a' is an integer

(B) There exists no real 'a'.

(C) There exists more than one value of 'a'

(D) Some possible value of 'a' are Irrational.

Ans. **(C, D)**

Sol. $a-(\ell nx)^{3/2} = t$

$-\frac{3}{2}\frac{(\ell nx)^{1/2}}{x}dx = dt$

$I = \frac{2}{3}\int_{a-1}^{a}\frac{dt}{t^2} = 1$

$-\frac{2}{3}\left(\frac{1}{t}\right)_{a-1}^{a} = 1$

$\frac{1}{a} - \frac{1}{a-1} = -\frac{3}{2}$

$\frac{1}{a(a-1)} = \frac{3}{2} \Rightarrow 2 = 3a^2 - 3a$

$3a^2 - 3a - 2 = 0$

12. Consider the parabola $y^2 = 4x$. Point P is (–2, 1) & S is its focus. Two tangents from point P touch the parabola at P_1 & P_2. Foot of perpendiculars are drawn from P on lines SP_1 & SP_2 and they are Q_1 & Q_2 respectively. Then

(A) $Q_1Q_2 = 3\sqrt{\frac{2}{5}}$ (B) $Q_1Q_2 = 2$ (C) $PQ_1 = 3$ (D) $PQ_1 = 4$

Ans. **(A,C)**

Sol. $y = mx + \frac{1}{m}$

$1 = -2m + \frac{1}{m}$

$m = -2m^2 + 1$

$2m^2 + m - 1 = 0$

$(2m - 1)(m + 1) = 0$

$m_1 = \frac{1}{2}, m_2 = -1$

$P_1(4, 4), P_2(1, -2)$

equation of $P_1 S \Rightarrow y = \frac{4}{3}(x-1) \Rightarrow 4x - 3y - 4 = 0$

equation of $P_2S \Rightarrow x = 1$

for $Q_1(\alpha, \beta)$

$$\frac{\alpha+2}{4} = \frac{\beta-1}{-3} = -1\left\{\frac{-8-3-4}{25}\right\}$$

$$\alpha = \frac{2}{5},\ \beta = -\frac{4}{5} \Rightarrow Q_1\left(\frac{2}{5}, -\frac{4}{5}\right) \text{ and } Q_2(1, 1)$$

$$Q_1Q_2 = \sqrt{\frac{9}{25}+\frac{81}{25}} = 3\sqrt{2/5}$$

$$SP_2 = 2$$

$$PQ_1 = \sqrt{\frac{144}{25}+\frac{81}{25}} = 3$$

13. Consider $f(n) - n = \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \frac{48-3n-3n^2}{12n+3n^2} + \ldots\ldots$ (upto n terms), then $\lim_{n\to\infty} f(n)$ is

(A) $4 - \frac{3}{4}\ell n\frac{7}{3}$ (B) $4 + \frac{3}{4}\ell n\frac{5}{4}$

(C) $2 - \frac{5}{4}\ell n\frac{4}{3}$ (D) $3 + \frac{5}{4}\ell n\frac{5}{3}$

Ans. (A)

Sol. $$f(n) = n + \sum_{r=1}^{n}\frac{16r+(9-4r)n-3n^2}{4rn+3n^2}$$

$$f(n) = n + \sum_{r=1}^{n}\frac{(16r+9n)-(4rn+3n^2)}{(4rn+3n^2)}$$

$$f(n) = n + \sum_{r=1}^{n}\left\{\frac{16r+9n}{4rn+3n^2}-1\right\} = \sum_{r=1}^{n}\frac{16r+9n}{4rn+3n^2}$$

$$\lim_{n\to\infty} f(n) = \int_0^1 \frac{16x+9}{4x+3}dx = \int_0^1 \frac{4(4x+3)-3}{(4x+3)}dx = \left\{4x-\frac{3}{4}\ell n(4x+3)\right\}_0^1 = \left(4-\frac{3}{4}\ell n\frac{7}{3}\right)$$

14. $B = \lim_{x\to 0}\frac{e^{x^3}-(1-x^3)^{1/3}+((1-x^2)^{1/2}-1)\sin x}{x\sin^2 x}$ find 6B

Ans. (5)

Sol. $$B = \lim_{x\to 0}\frac{1+x^3-\left(1-\frac{x^3}{3}\right)+\left(1-\frac{x^2}{2}-1\right)(x)}{x^3\left(\frac{\sin x}{x}\right)^2}$$

$$B = \lim_{x\to 0}\frac{\left(1+\frac{1}{3}-\frac{1}{2}\right)x^3}{x^3} = \frac{5}{6}$$

$$\Rightarrow 6B = 5$$

15. Consider 4 boxes having 3 Red & 2 Blue balls each. All boxes & balls are distinct. Find the number of ways in which we can choose 10 balls such that atleast 1 Red & 1 Blue ball is selected from each box.

(A) 20806 (B) 21816 (C) 21806 (D) 20816

Ans. (B)

Sol.

	B_1(3R + 2B)	B_2(3R + 2B)	B_3(3R + 2B)	B_4(3R + 2B)
(1)	3R,1B	1R,1B	1R,1B	1R,1B
(2)	2R,1B	2R,1B	1R,1B	1R, 1B
(3)	1R, 2B	1R,2B	1R, 1B	1R, 1B
(4)	2R, 2B	1R,1B	1R, 1B	1R, 1B
(5)	2R, 1B	1R,2B	1R, 1B	1R, 1B

Total ways = ${}^4C_1.\,[{}^3C_3.{}^2C_1.({}^3C_1{}^2C_1)^3] + {}^4C_2\,[({}^3C_2{}^2C_1)^2\,({}^3C_1{}^2C_1)^2]$

$+\ {}^4C_2.\,[({}^3C_1{}^2C_2)^2({}^3C_1{}^2C_1)^2] + {}^4C_1\,[({}^3C_2{}^2C_2)\,({}^3C_1{}^2C_1)^2]$

$+\ {}^4C_2\,\underline{|2}\ [{}^3C_2{}^2C_1.{}^3C_1{}^2C_2({}^3C_1{}^2C_1)^2]$

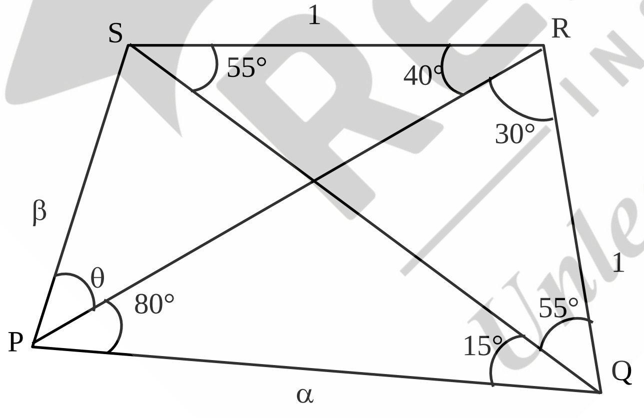
= 21816

16. Let PQRS is a quadrilateral whose $\angle PQR = 70° = \angle QRS$. Also $\angle PRS = 40°$ & $\angle PQS = 15°$. $\angle RPS = \theta$ & $QR = 1$. If $PS = \beta$ & $PQ = \alpha$, Then $4\alpha\beta\sin\theta$ lies in -

(A) $\left(\sqrt{2}, 2\right)$ (B) $\left(0, \sqrt{2}\right)$ (C) $\left(1, \sqrt{2}\right)$ (D) (1, 3)

Ans. **(B,C,D)**

Sol.



In ΔPQR $\dfrac{\alpha}{\sin 30°} = \dfrac{1}{\sin 80°}$ $\Rightarrow \alpha = \dfrac{1}{2\sin 80°}$

In ΔPRS $\dfrac{\beta}{\sin 40°} = \dfrac{1}{\sin\theta}$

$\Rightarrow \beta\sin\theta = \sin 40°$

$\therefore\ 4\alpha\beta\sin\theta = \dfrac{2}{\sin 80°}\sin 40° = \sec 40°$

$\therefore\ \sec 30° < \sec 40° < \sec 45°$

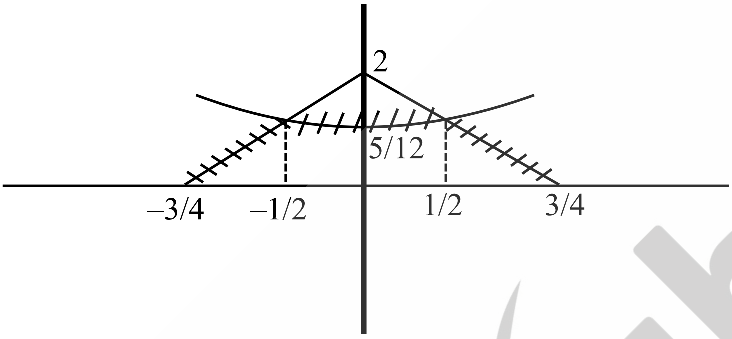
$\Rightarrow \sec 40° \in \left(\dfrac{2}{\sqrt{3}}, \sqrt{2}\right)$

17. Consider $f(x) = x^2 + \frac{5}{12}$ & $g(x) = \begin{cases} 2\left(1-\frac{4|x|}{3}\right) & ; \quad |x| \le \frac{3}{4} \\ 0 & ; \quad |x| > \frac{3}{4} \end{cases}$ find the area

$$9\left\{(x,y): RxR \mid x| \le \frac{3}{4}, \min\left(f(x), g(x)\right)\right\}$$

Ans. **(6)**

Sol.



$y = x^2 + \frac{5}{12}$, $y = \frac{2-8x}{3}$

$\frac{2-8x}{3} = x^2 + \frac{5}{12}$

$x^2 + \frac{8x}{3} + \frac{5}{12} - 2 = 0$

$12x^2 + 32x - 19 = 0$

$12x^2 + 38x - 6x - 19 = 0$

$2x(6x + 19) - 1\,(6x + 19) = 0$

$(6x + 19)\,(2x - 1) = 0$

$x = \frac{-19}{6}, \frac{1}{2}$

$$\text{Area} = 2\left\{\int_0^{\frac{1}{2}}\left(x^2 + \frac{5}{12}\right)dx + \int_{\frac{1}{2}}^{\frac{3}{4}}\left(2 - \frac{8x}{3}\right)dx\right\}$$

$$= 2\left\{\left(\frac{x^3}{3} + \frac{5x}{12}\right)_0^{\frac{1}{2}} + \left(2x - \frac{4x^2}{3}\right)_{\frac{1}{2}}^{\frac{3}{4}}\right\}$$

$$= 2\left\{\left(\frac{1}{24} + \frac{5}{24}\right) + \left(\frac{3}{2} - \frac{3}{4}\right) - \left(1 - \frac{1}{3}\right)\right\}$$

$$A = 2\left\{\frac{1}{4} + \frac{3}{4} - \frac{2}{3}\right\} = \frac{2}{3}$$

$9A = 6$

18. If $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$ and $\sin(\alpha+\beta) = \frac{1}{3}$, also $\cos(\alpha-\beta) = \frac{2}{3}$.

Then find $\left(\frac{\sin\alpha}{\cos\beta}+\frac{\cos\beta}{\sin\alpha}+\frac{\cos\alpha}{\sin\beta}+\frac{\sin\beta}{\cos\alpha}\right)^2$

Sol. $S = \left(\frac{\cos(\alpha-\beta)}{\cos\beta\sin\beta}+\frac{\cos(\alpha-\beta)}{\sin\alpha\cos\alpha}\right)^2$

$S = \frac{4}{9}\left(\frac{\frac{2}{9}}{\sin\alpha\cos\alpha\cos\beta\sin\beta}\right)^2$

$S = \frac{4}{81}\cdot\frac{4}{9}\left(\frac{1}{\sin\alpha\cos\alpha\cos\beta\sin\beta}\right)^2$

Now,

$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{1}{3}$

$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{2}{3}$

square & Add, $\cos^2\beta + \sin^2\beta + 4\sin\alpha\cos\alpha\sin\beta\cos\beta = \frac{5}{9}$

$\Rightarrow \quad \sin\alpha\cos\alpha\sin\beta\cos\beta = -\frac{1}{9}$

$\therefore \quad S = \frac{16}{9}$

19. Consider the curve : x = –t, y = t – p & z = p + 1. If S is a point (10,15,20) whose reflection in the curve is Q(α,β,γ) then :

(A) $3(\alpha+\beta+\gamma) = -121$ (B) $3(\alpha+\beta) = -101$

(C) $3(\beta+\gamma) = -71$ (D) $3(\gamma+\alpha) = -86$

Ans. **(B,C,D)**

Sol. Curve is x + y + z = 1

Now $\frac{\alpha-10}{1} = \frac{\beta-15}{1} = \frac{\gamma-20}{1} = -2\left(\frac{10+15+20-1}{3}\right) = \frac{-88}{3}$

$\alpha = \frac{-58}{3}, \beta = -\frac{43}{3}, \gamma = -\frac{28}{3}$

20. $\frac{dy}{dx} + 12y = \cos\left(\frac{\pi x}{12}\right)$ and y(0) = 0 then :

(A) 'y' is an increasing function

(B) 'y' is a decreasing function

(C) 'y' is a non-periodic function

(D) There exists y = b for which there exists infinite solution of 'x'

Ans. **(C,D)**

Sol. $y.e^{12x} = \int \cos\frac{\pi x}{12} \times e^{12x}\,dx$

$$y.e^{12x} = \cos\frac{\pi x}{12}\times\frac{e^{12x}}{12}+\frac{\pi}{12}\int \sin\frac{\pi x}{12}\times\frac{e^{12x}}{12}dx$$

$$I = \cos\frac{\pi x}{12} \times \frac{e^{12x}}{12} + \frac{\pi}{144}\left\{\sin\frac{\pi x}{12}\times\frac{e^{12x}}{12}-\frac{\pi}{144}\int\cos\frac{\pi x}{12}\times e^{12x}dx\right\}$$

$$= \cos\frac{\pi x}{12}\times\frac{e^{12x}}{12}+\frac{\pi}{144}\left\{\sin\frac{\pi x}{12}\times\frac{e^{12x}}{12}-\frac{\pi}{144}I\right\}$$

$$= \frac{1}{12}\times\cos\frac{\pi x}{12}e^{12x}+\frac{\pi}{144}\times\frac{1}{12}e^{12x}\sin\frac{\pi x}{12}-\left(\frac{\pi}{144}\right)^2 I$$

$$I\left(1+\frac{\pi^2}{(144)^2}\right) = \frac{e^{12x}}{12}\left(\cos\frac{\pi x}{12}+\frac{\pi}{144}\sin\frac{\pi x}{12}\right)$$

$$I = \frac{e^{12x}\times 144\times 12}{(144)^2+\pi^2}\left(\cos\frac{\pi x}{12}+\frac{\pi}{144}\times\sin\frac{\pi x}{12}\right)$$

$$y = \frac{144\times 12}{\pi^2+(144)^2}\left(\cos\frac{\pi x}{12}+\frac{\pi}{144}\times\sin\frac{\pi x}{12}\right)+C\,e^{-12x}$$

neither increasing nor decreasing

21. Two player P_1, P_2 are throwing a dice starting with P_1. If x_i , y_i be the sum of results obtained by P_1, P_2 respectively in i^{th} throw. If in any particular throw score of P_1 is greater, equal or less than P_2 then P_1 gets 5, 2 or 0 points respectively, then :

(A) $P(x_2 \geq y_2)$	(P) $\frac{11}{16}$
(B) $P(x_2 > y_2)$	(Q) $\frac{509}{864}$
(C) $P(x_3 \geq y_3)$	(R) $\frac{5}{16}$
(D) $P(x_3 > y_3)$	(S) $\frac{355}{864}$

Ans. **(A-P, B-R, C-Q, D-S)**

Sol. $P(x_2 = y_2) + P(x_2 > y_2) + P(x_2 < y_2) = 1$

$$\Rightarrow \left(\left[\frac{1}{6}\right]^2+\left(\frac{^6C_2}{36}\right)^2\times 2!\right)+k+k=1$$

(Here let $P(x_2 > y_2) = P(x_2 < y_2) = k$)

$$\Rightarrow \frac{3}{8}+2k=1$$

$$\therefore \quad k=\frac{5}{16}$$

$\therefore \quad P(x_2 > y_2) = \dfrac{5}{16}$

$P(x_2 \geq y_2) = \dfrac{5}{16} + \dfrac{3}{8} = \dfrac{11}{16}$

$P(x_3 = y_3) + P(x_3 > y_3) + P(x_3 < y_3) = 1$

$P(x_3 = y_3) = P(3\text{ Draws}) + P(1W, 1L, 1D) = \left(\dfrac{1}{6}\right)^3 + 3!\,.\left(\dfrac{15}{36}.\dfrac{15}{36}.\dfrac{1}{6}\right)$

$= \dfrac{1}{216} + \dfrac{5}{12} \times \dfrac{5}{12} = \dfrac{2+75}{36\times 12} = \dfrac{77}{36\times 12}$

$\therefore \quad 2P(x_3 > y_3) = 1 - \dfrac{77}{432}$

$\Rightarrow \quad P(x_3 > y_3) = \dfrac{355}{864}$

$\therefore \quad P(x_3 \geq y_3) = \dfrac{355}{864} + \dfrac{154}{864} = \dfrac{509}{864}$

22. |M| denotes det. of square matrix M. Let $g : \left[0, \dfrac{\pi}{2}\right] \to R$, $g(\theta) = \sqrt{f(\theta)-1} + \sqrt{f\left(\dfrac{\pi}{2}-\theta\right)-1}$,

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta+\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right) \\ \sin\left(\theta-\frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta+\frac{\pi}{4}\right) & \log_e\frac{\pi}{4} & \tan\pi \end{vmatrix}$$

Let p(x) be 2 degree polynomial whose roots are min and max of $g(\theta)$, $p(2) = 2\sqrt{2}$ then

(A) $p\left(\dfrac{3+\sqrt{2}}{4}\right) < 0$

(B) $p\left(\dfrac{5-\sqrt{2}}{4}\right) < 0$

(C) $p\left(\dfrac{1+3\sqrt{2}}{4}\right) > 0$

(D) $p\left(\dfrac{5\sqrt{2}-1}{4}\right) > 0$

Ans. **(A,B,D)**

Sol. $f(\theta) = \dfrac{1}{2}(2+2\sin^2\theta) + |A|$

$f(\theta) = 1 + \sin^2\theta$ {$|A| = 0 \because$ A is skew symm.}

$g(\theta) = \sqrt{f(\theta)-1} + \sqrt{f\left(\dfrac{\pi}{2}-\theta\right)-1}$

$g(\theta) = \sin\theta + \cos\theta$

$g(\theta)_{min} = -\sqrt{2}$

$g(\theta)_{max} = \sqrt{2}$

$P(x) = k(x^2 - 2)$

$\because P(2) = 2\sqrt{2}$